Full wave simulation of LH waves based on finite element method

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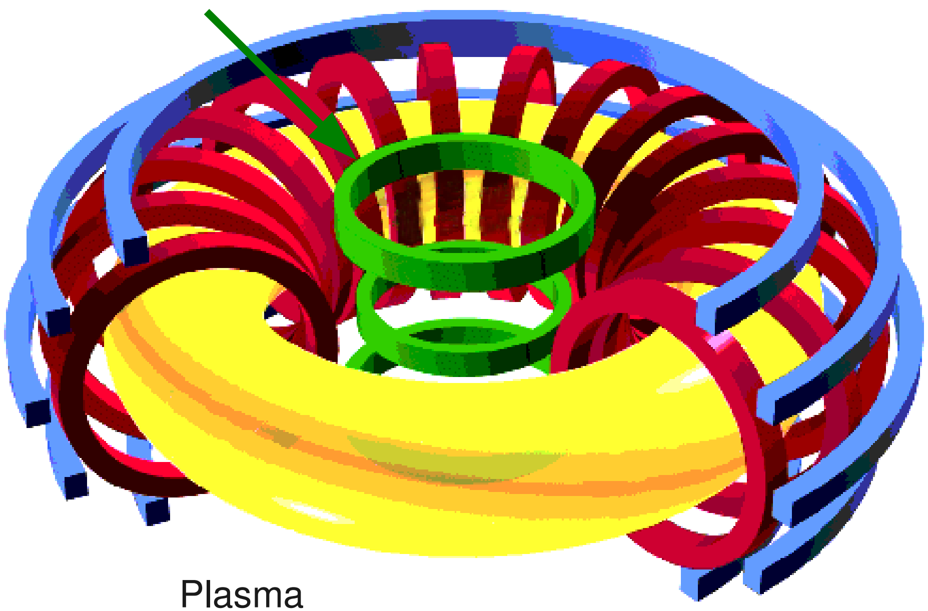
Tokamak experiments

- Confinement via closed magnetic field lines (toroidal topology)
  - Charged particles (e-, i+) follow the magnetic field lines
- Helical field is needed for confinement
  - Current flowing in the plasma is induced with the transformer principle
  - Inherently a pulsed device!

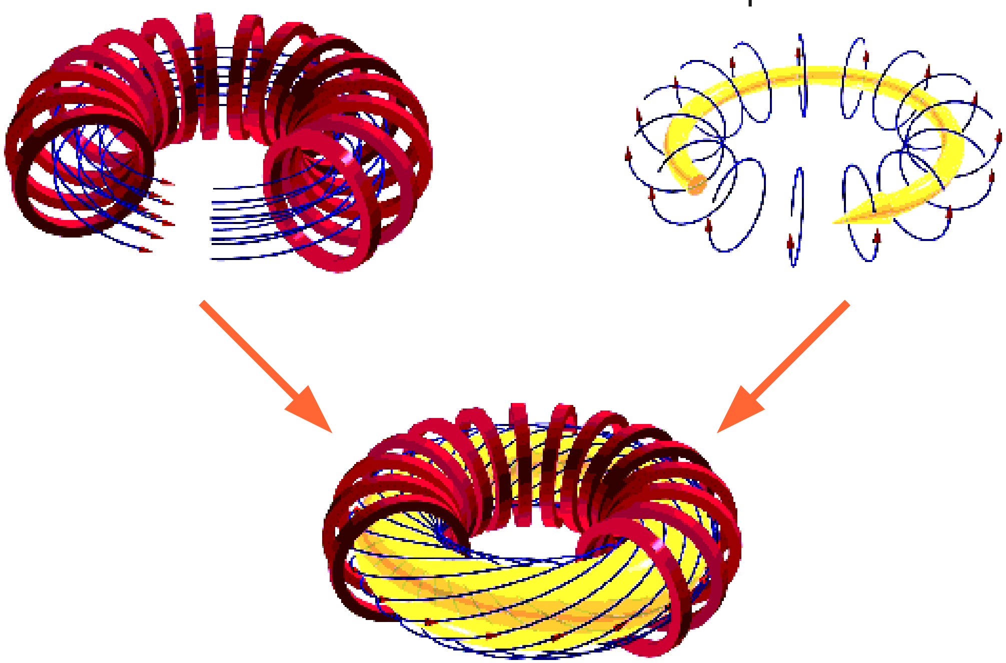
Ohmic coils
(Transformer primary circuit)



Plasma
(Transformer secondary circuit)

Toroidal field coils
and toroidal field

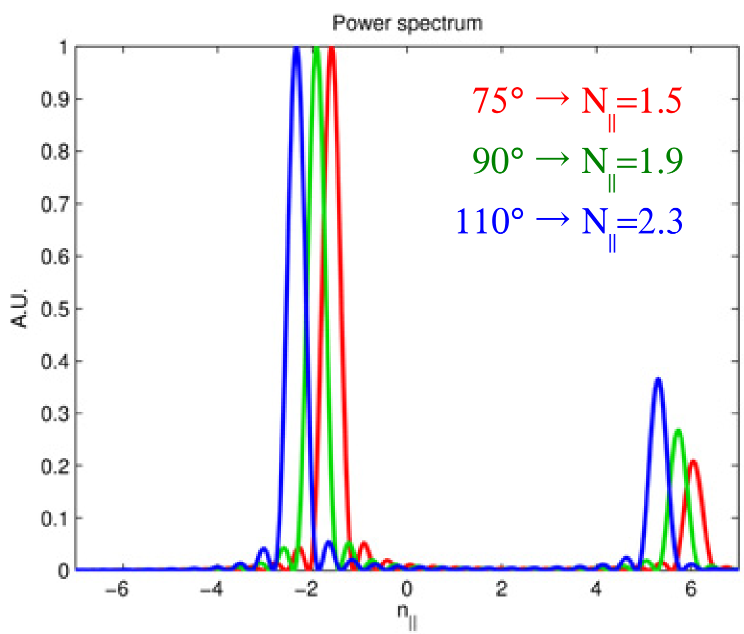
Plasma current
And poloidal field

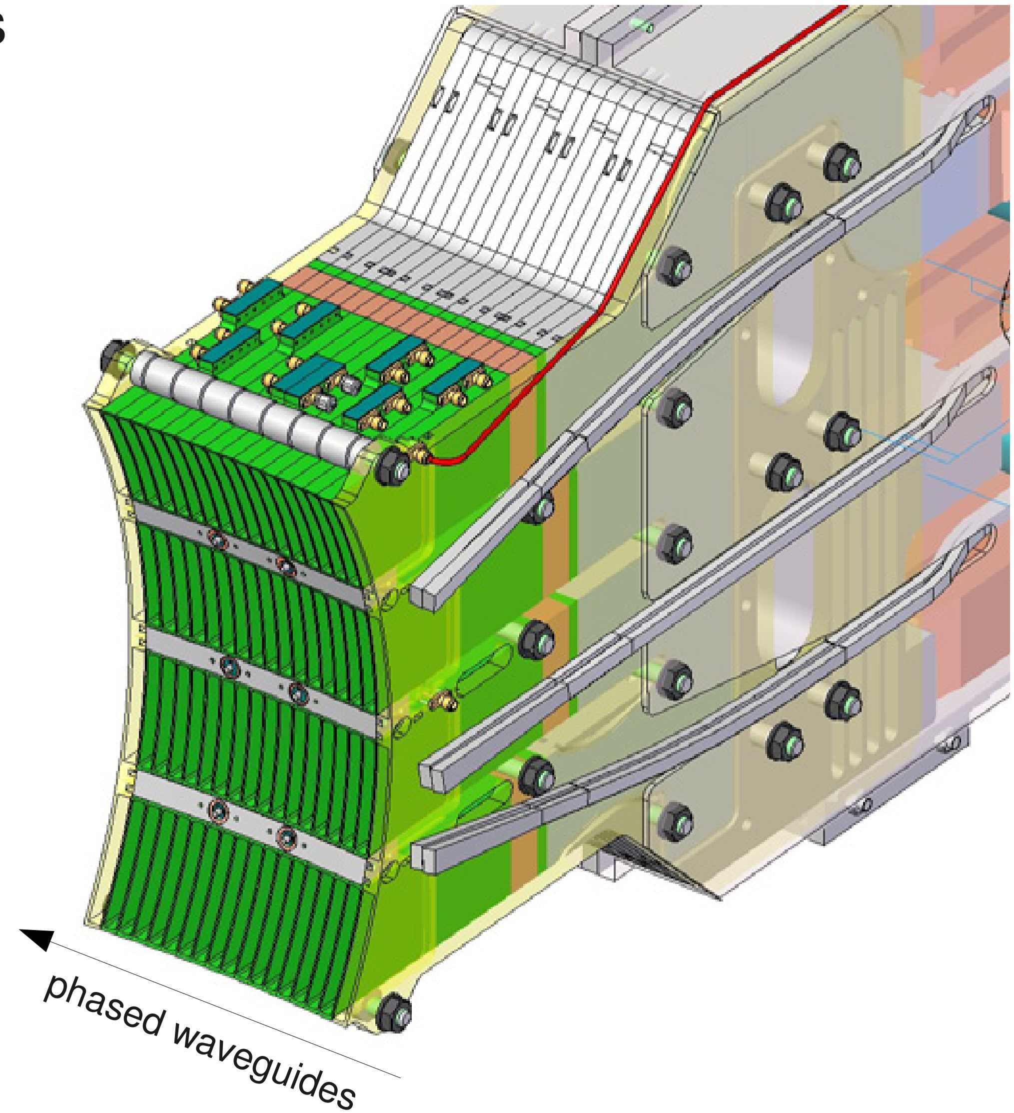


Total magnetic helical field

LH launcher on Alcator C-Mod

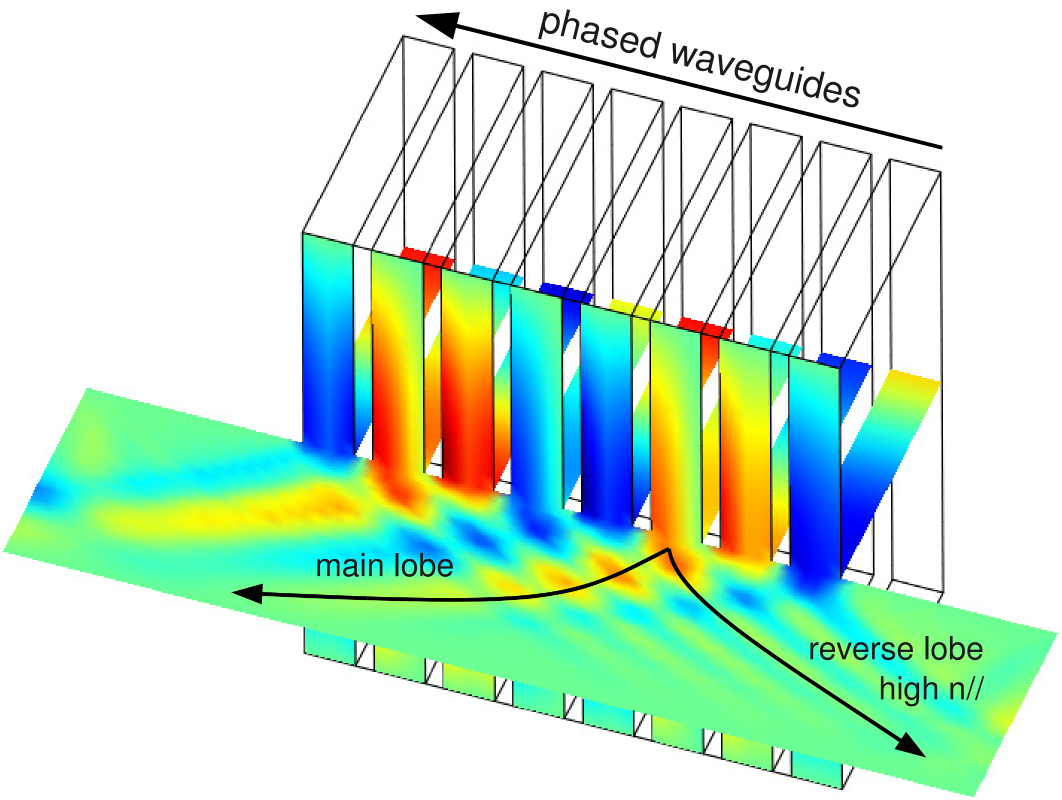
- Slow wave launching structures
  - Grill of phased waveguide
  - Traveling wave structures
- Upcoming LH system
  - 16x4 waveguides
  - 60mm X 7mm
  - 1.4MW @ 4.6GHz

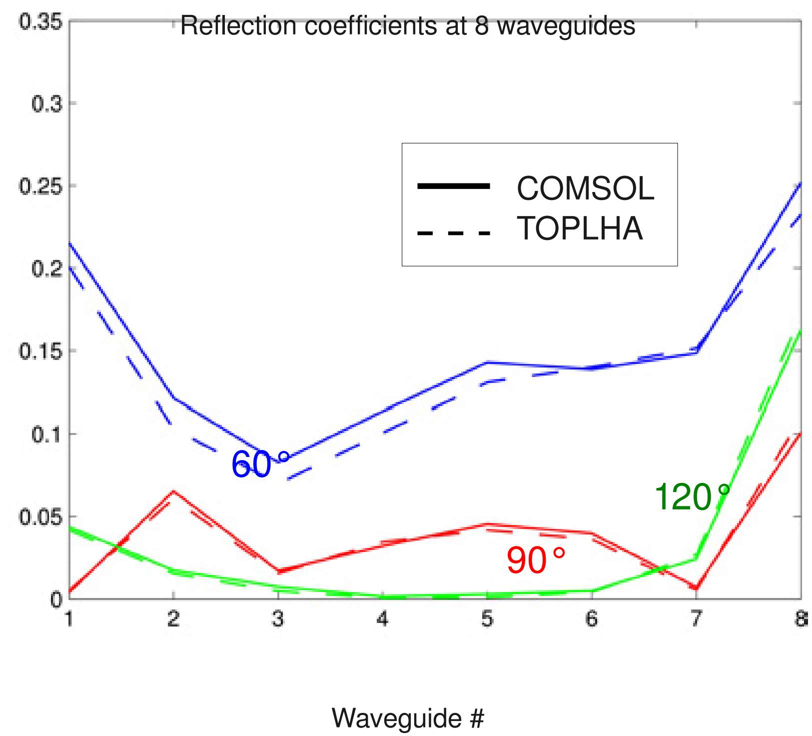




3D cold plasma modeling

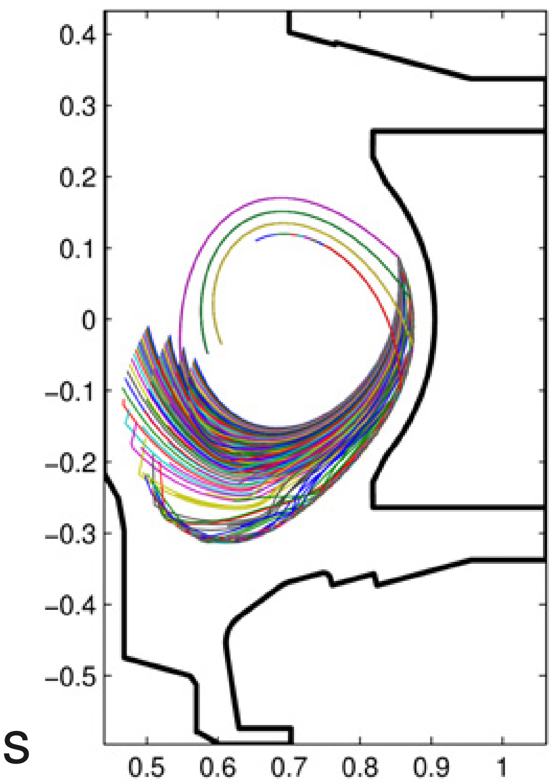
- Cold plasma simulations with COMSOL have been extensively verified
  - e.g. by comparing coupling coefficients versus TOPLHA code





Motivations

- Modeling LH by ray tracing has several long-standing issues
  - WKB requires ΔK/K<<1 which for LH waves in Tokamak plasmas is questionable...
    - at low densities (small Kperp)
    - in fast changing density (big ΔK)
    - near cutoffs ($P \rightarrow 0$)
    - near caustics ($|K_R| \rightarrow 0$)
  - Ambiguity in the launched spectrum
    - Ray has to start inside the cutoff
    - Finite height of waveguide

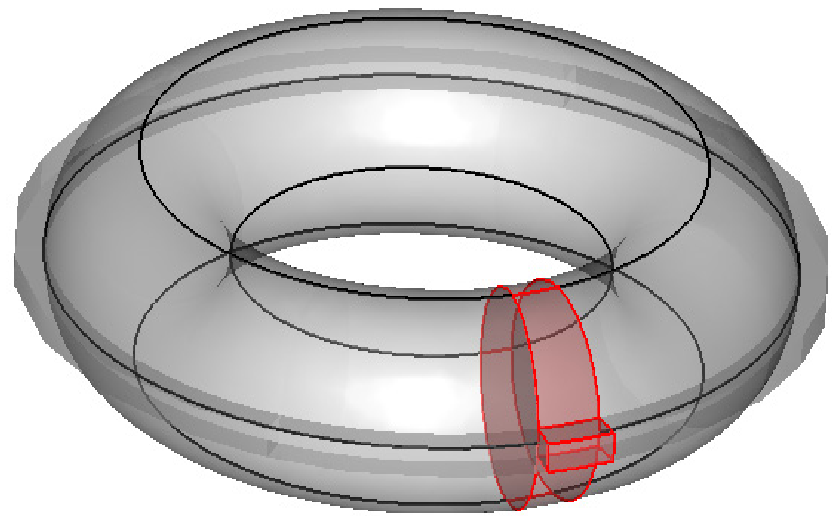


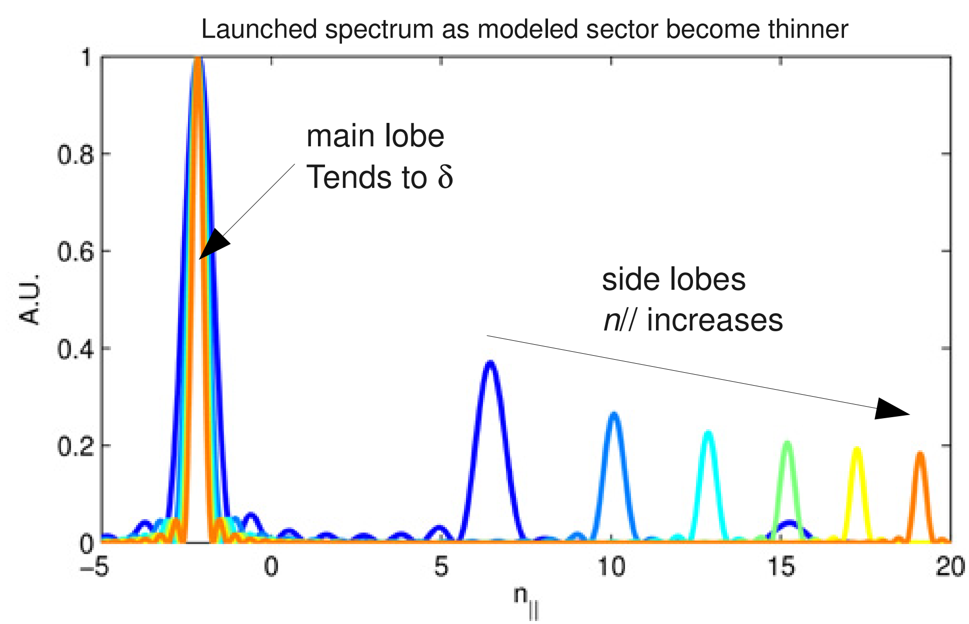
GENRAY simulation by G. Wallace

- There are two approaches to full wave simulations
  - Wave-number domain approach (e.g. TORIC, AORSA)
  - Real space domain... what I am going to present
- A full-wave 3D calculation of the whole torus is still too computationally demanding... (λ~1mm, plasma size~1m^3)

Single toroidal mode analysis

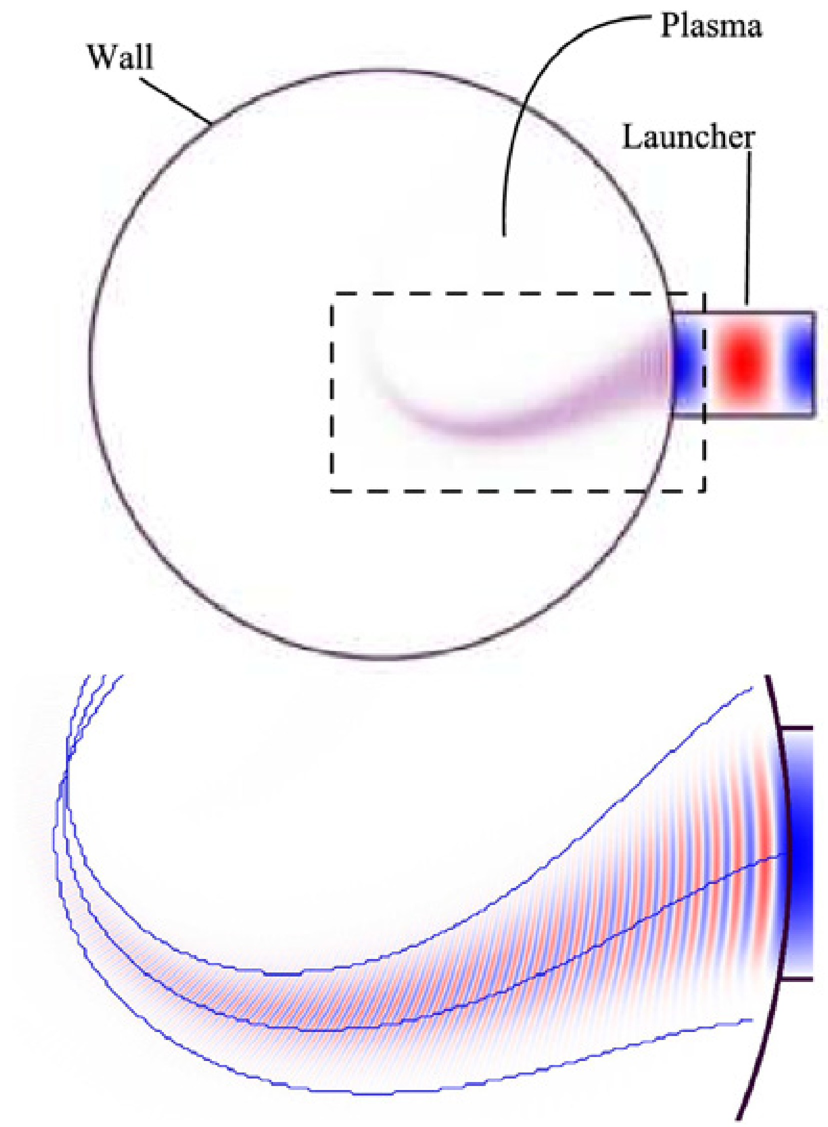
- Launching waves from an infinite number of infinitesimally thin phased waveguides
  - Spectrum is a δ function at given $n_{||}$
- Exploit this idea to do single toroidal mode decomposition in a 3D FEM solver
  - Model a single toroidal section having finite thickness
  - Periodic boundary condition at the sides of the toroidal slice
    - Phase relation between the solution on the sides of slice determines $n_{||}$
  - Spectrum approaches a δ function as thickness → 0





Example of single toroidal mode analysis

- Alcator C plasma
  - a=0.17 [m]
  - R_0=0.64 [m]
  - f = 4.6 [GHz]
  - $n_{||}$ = 2.5
  - B_0 = 8 [T]
  - Parabolic profiles
    - n_{e0} = 5E19[m^{-3}]
    - I_P = 400 [KA]
- Wave damping is introduced through collisions
- ELD is necessary for correct evaluation of wave damping



Comparison of full wave electric field and ray tracing trajectory

LH waves in a Maxwellian plasma

- Algebraic equation in the wavenumber domain

$$\vec{k}\times\left(\vec{k}\times\vec{E}(\vec{k})\right)+\frac{\omega^2}{c^2}\boxed{\bar{\bar{\varepsilon}}_{\mathrm{LH}}(\vec{k})\cdot\vec{E}(\vec{k})}=0$$

Product

  - LH dielectric tensor: cold plasma + electron Landau damping

$$\bar{\bar{\varepsilon}}_{\mathrm{LH}}=\bar{\bar{\varepsilon}}_{cold}-i\bar{\bar{\varepsilon}}_{\mathrm{L}}=\begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}-i\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_{\mathrm{L}}(k_z) \end{pmatrix}$$

$$\varepsilon_{\mathrm{L}}(k_z)=\sqrt{\pi}\frac{\omega_{\mathrm{pe}}^2\omega}{|k_z|^3 v_{\mathrm{th}}^3}\exp\left(-\frac{\omega^2}{k_z^2 v_{\mathrm{th}}^2}\right)$$ (Maxwellian)

- An integro-differential equation in real space

$$\nabla\times(\nabla\times\vec{E}(\vec{x}))+\frac{\omega^2}{c^2}\left(\bar{\bar{\varepsilon}}_{cold}\cdot\vec{E}(\vec{x})-i\frac{\hat{z}}{\sqrt{2\pi}}\boxed{\int\varepsilon_{\mathrm{L}}(z-z')E_z(z')dz'}\right)=0$$

Convolution integral

$$\varepsilon_{\mathrm{L}}(z)=\frac{1}{\sqrt{2\pi}}\int\varepsilon_{\mathrm{L}}(k_z)e^{-ik_z z}dk_z$$

Fourier transform

Solution of the integro-differential equation

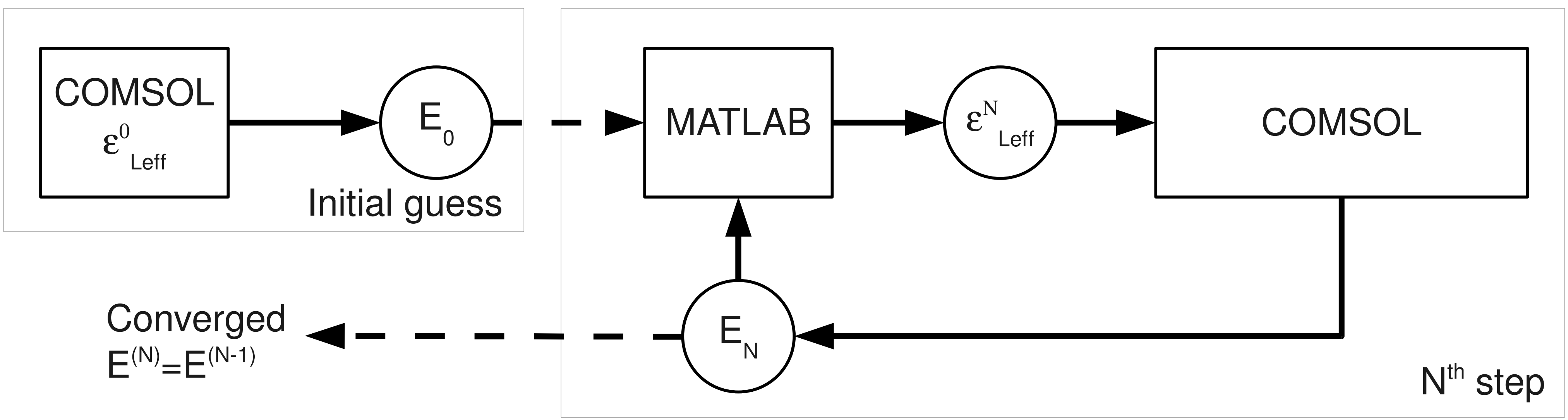
- Problem is split into two parts and solved iteratively

$$\nabla \times (\nabla \times \vec{E}^{N}(\vec{x})) + \frac{\omega^2}{c^2}\left(\bar{\bar{\varepsilon}}_{cold} - i\,\bar{\bar{\varepsilon}}_{Leff}\right) \cdot \vec{E}^{N}(\vec{x}) = 0$$

Conventional PDE which can be solved by COMSOL

$$\bar{\bar{\varepsilon}}^{(N)}_{Leff} = \frac{1}{E^{(N-1)}} \frac{\hat{z}}{\sqrt{2\pi}} \int \varepsilon_L(z-z')E_z^{(N-1)}(z')dz'$$

Convolution integral done in MATLAB



$$\cdots - i\,\frac{E^{(N)}(z)}{E^{(N-1)}(z)} \frac{1}{\sqrt{2\pi}} \int \varepsilon_L(z-z')E^{(N-1)}(z')dz' = 0$$

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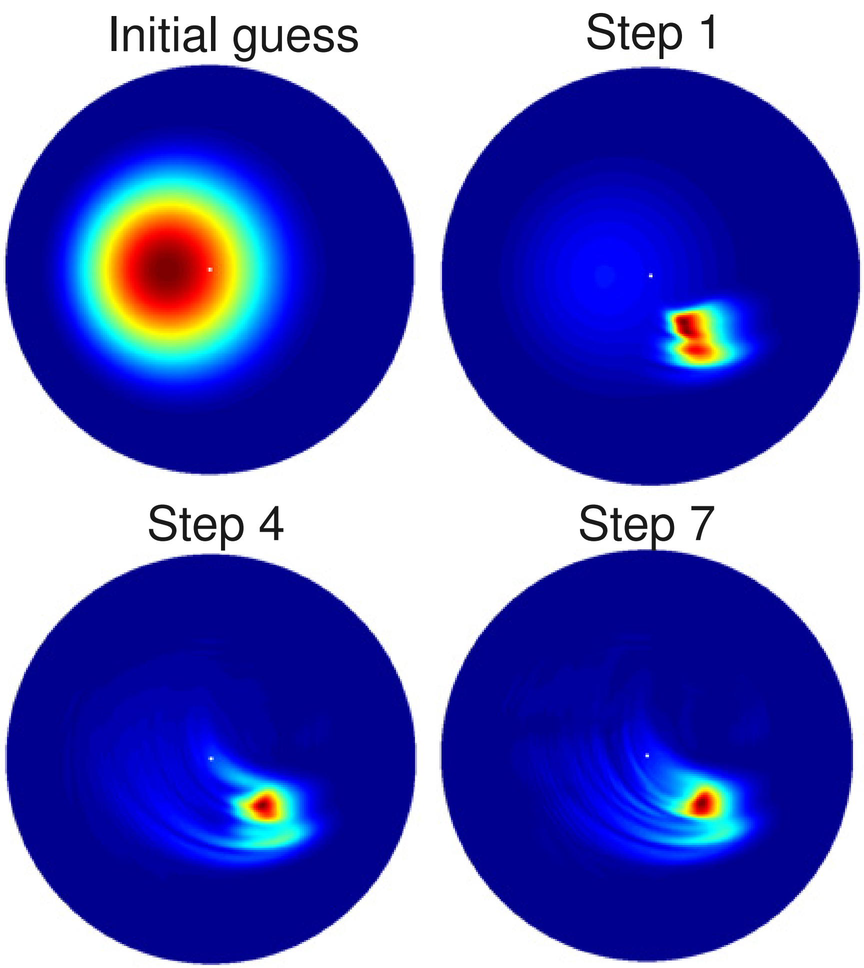
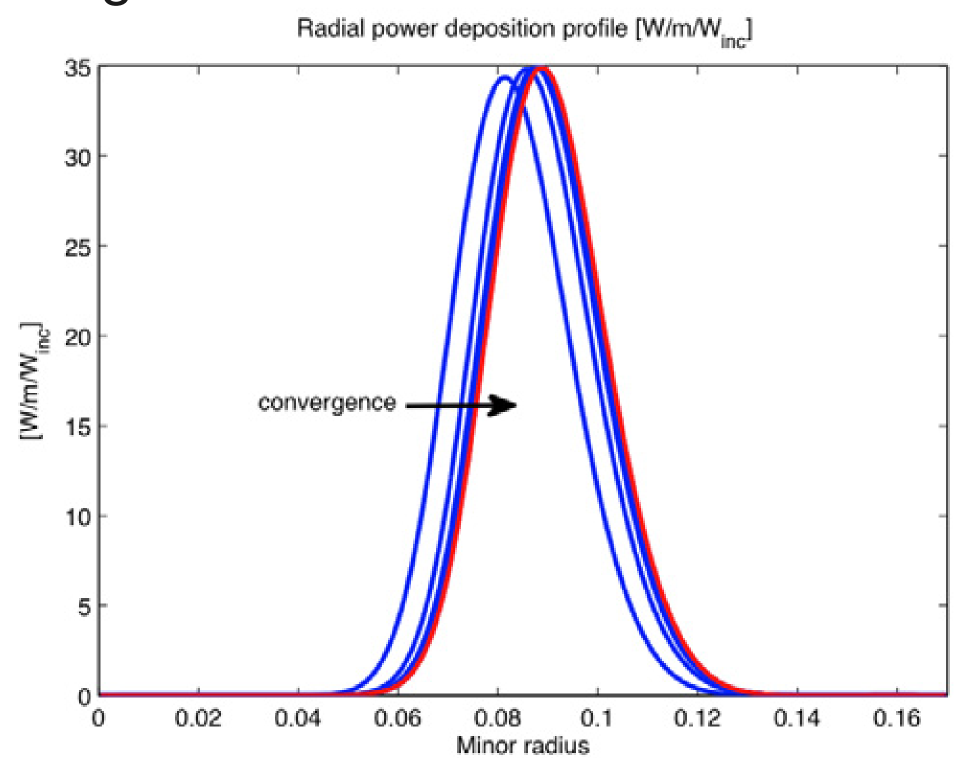
Convergence example



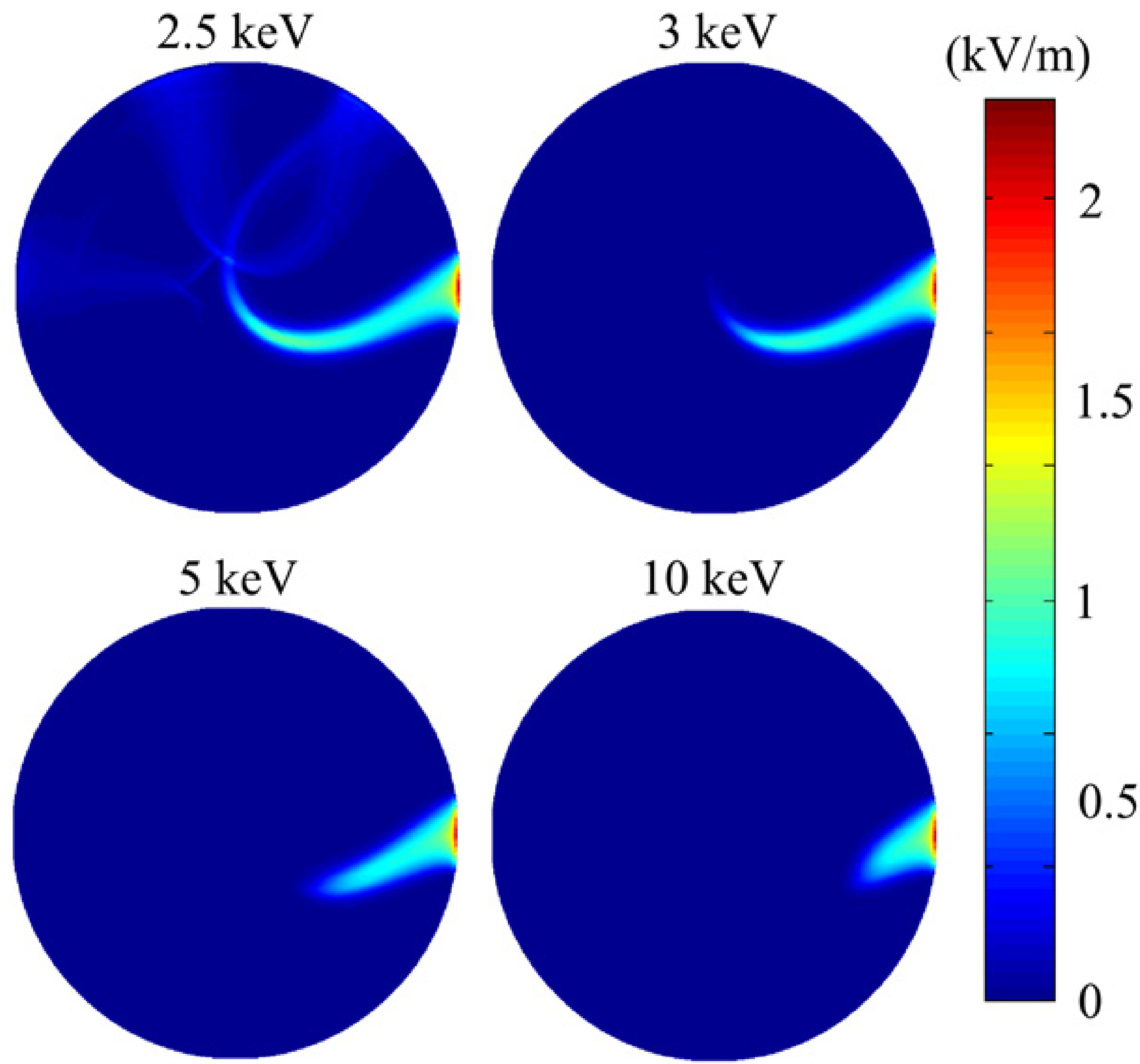
(~1M DoF, 10 mins for iteration)

- Effective damping ε_{Leff} and power absorption profile shows that solution converges in few steps
- Very robust with respect to initial guess

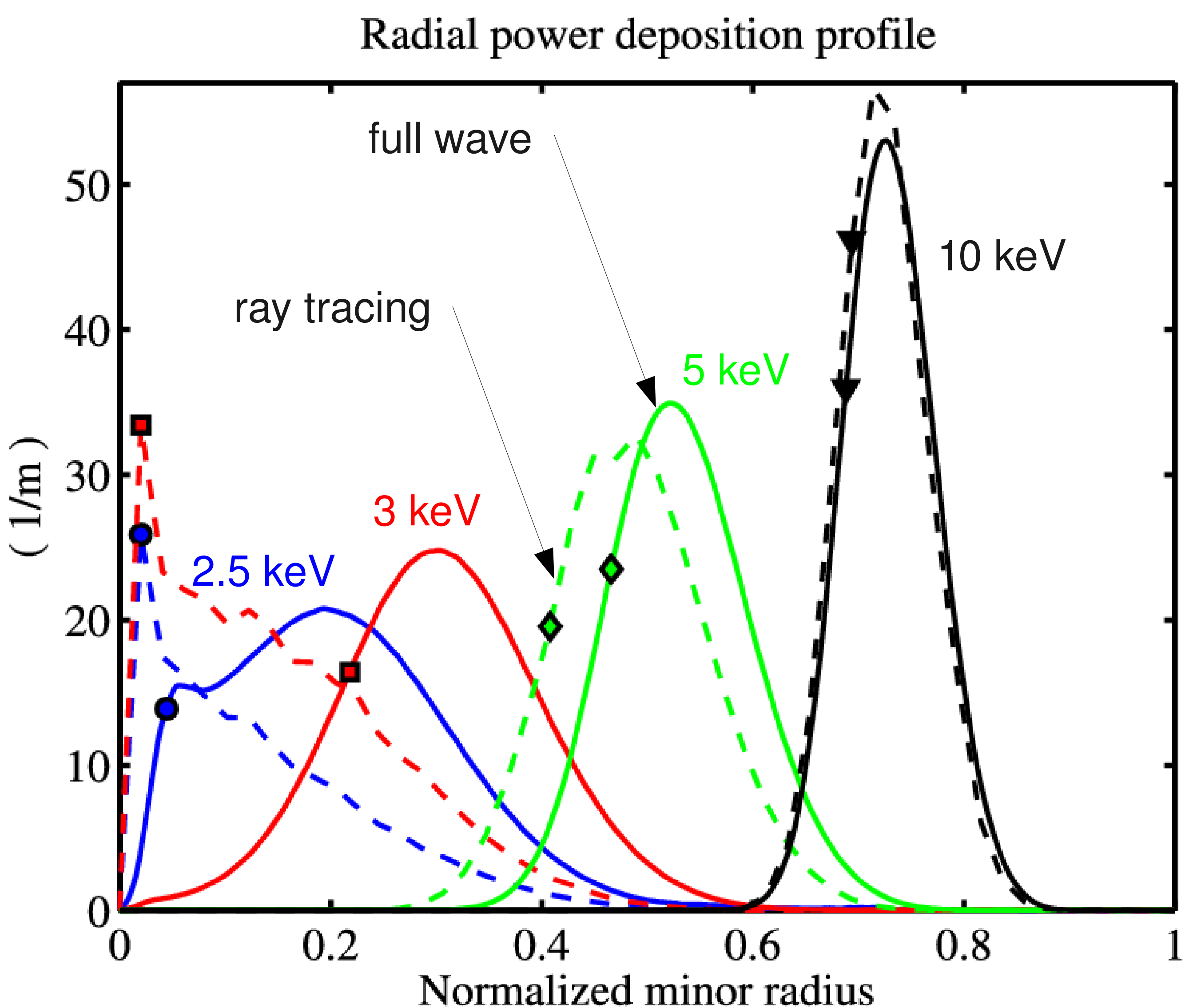


$|E_{||}|$ at different temperatures

- As temperature increases the wave penetration becomes shorter
  - Consistent: LH waves damp about where $v_{||}=\omega/k_{||}\sim 3\ vTe$
- At ~2.5 keV the propagation becomes multi-pass

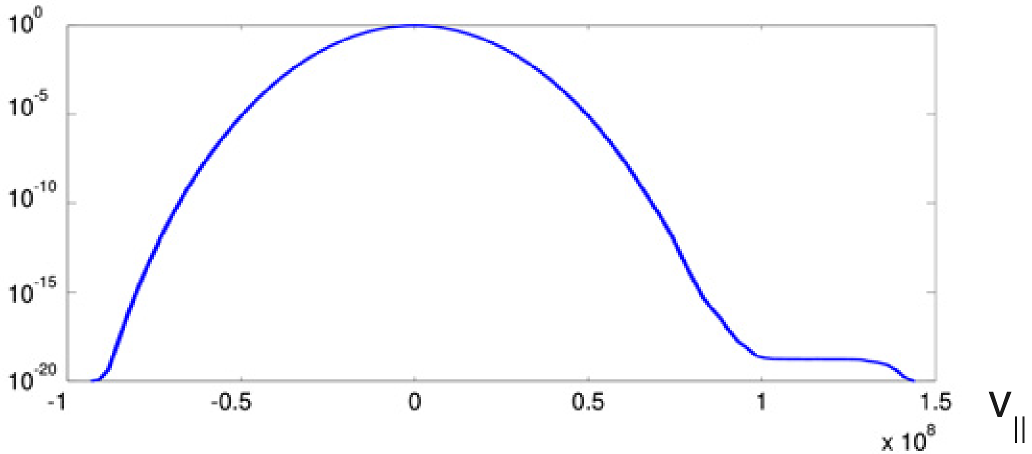


Radial power deposition profiles at different temperatures



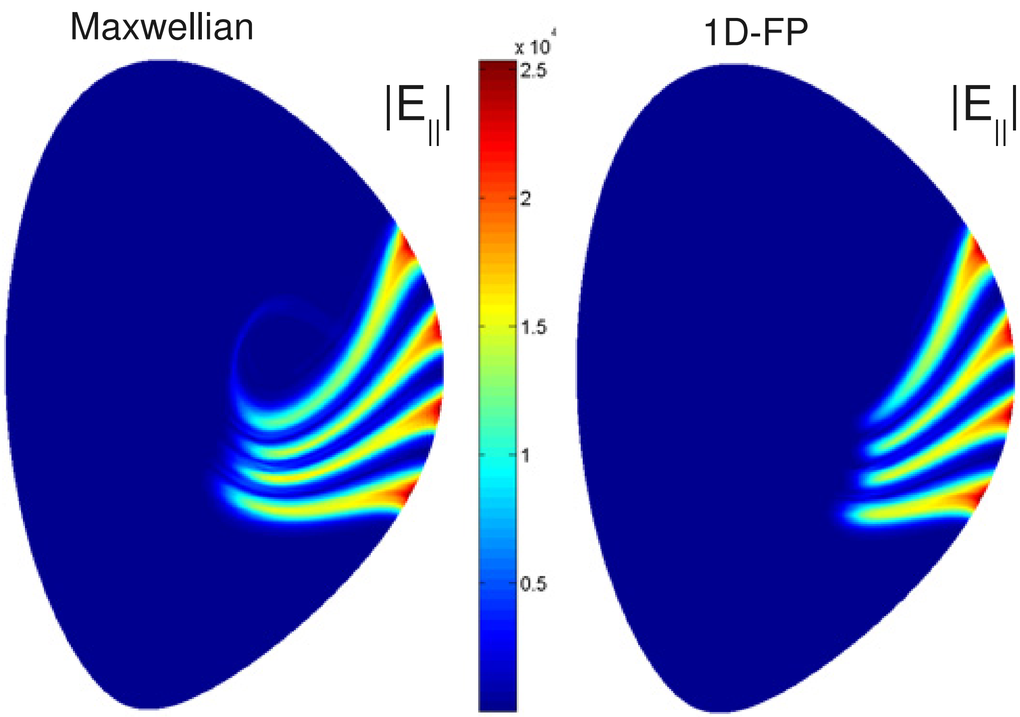
Integration with 1D Fokker Plank

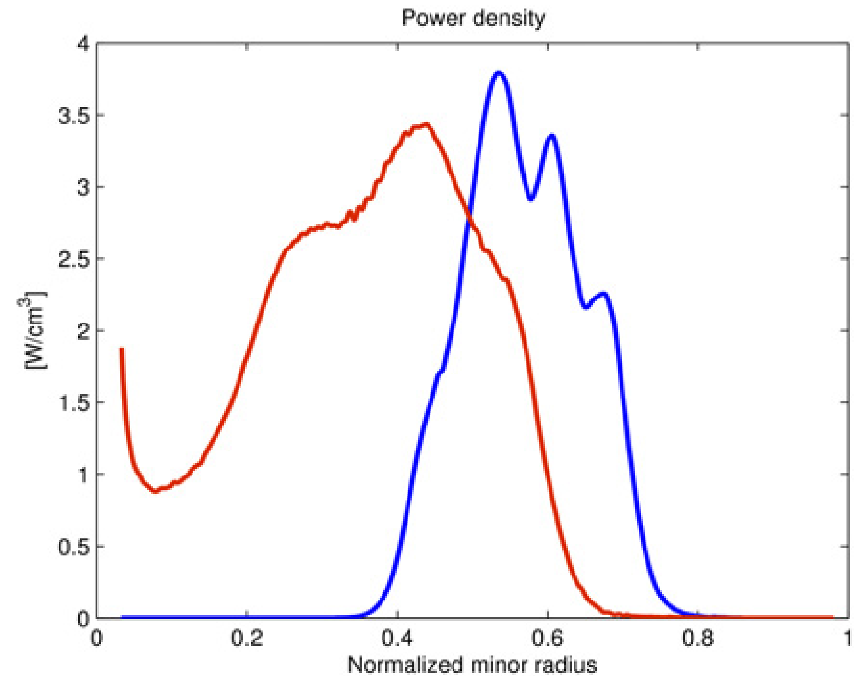
- Stationary solution for 1D FP equation
  - Wave fields distort the electrons velocity distribution, while collisions tend to restore Maxwellian
  - Formation of a tail, which changes the damping characteristics



- Parallel distribution function is evaluated at each step of the ELD iteration (Diffusion of distribution function due to RF fields)
  - Dielectric term $\varepsilon_L(k)$ is modified, and correspondingly ε_{Leff}
  - Hermitian part of the dielectric tensor is unchanged (the wave propagation is still described by the cold plasma propagation)

Integration with 1D FP shifts power deposition





- Alcator C-Mod
  - Equilibrium 1080320017
  - T_{e0} = 2.5 keV
  - $n_{e0} = 5\ 10^{19}\ m^{-3}$
  - $n_{||}$ = 2.5

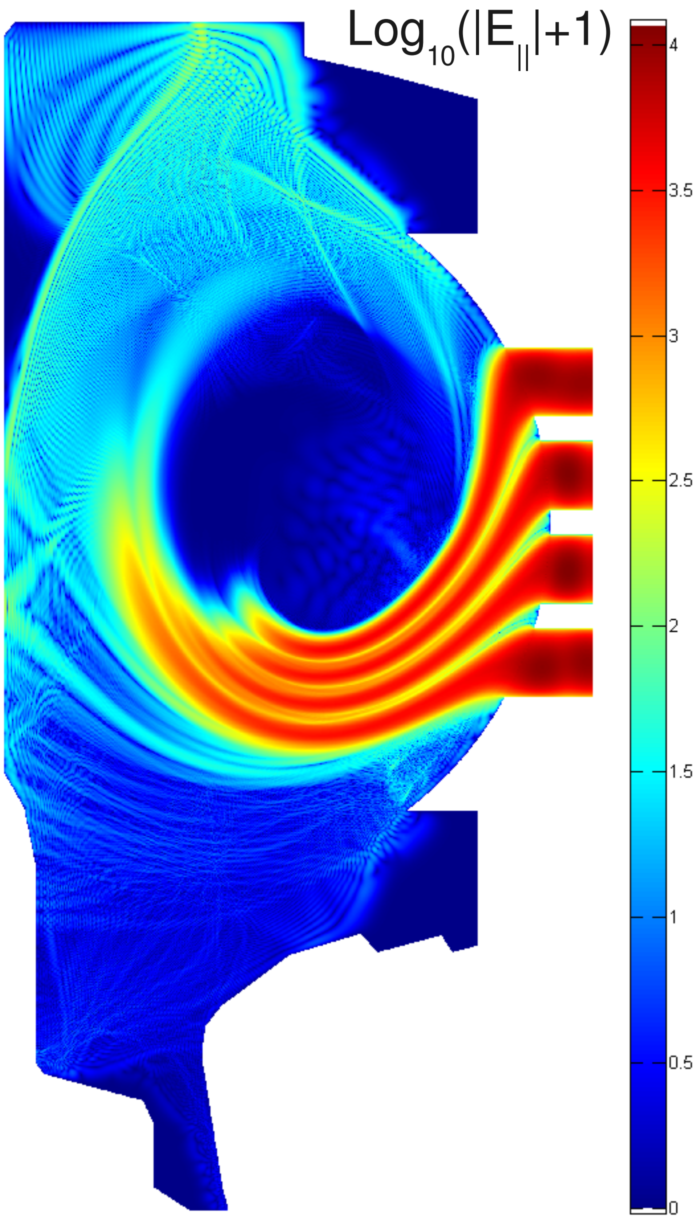
- Power deposition shifts outwards, consistently with larger population of fast electrons
- Convergence was not affected by the integration with 1D FP

Inclusion of SOL

- FEM approach allows seamless handling of antenna, first wall, SOL and core regions
  - LH waves propagate where $n_e > n_{e\_cutoff}$
- SOL modeled exponential decay as a function of magnetic flux topology
- Collisional damping by finite σ

$$\nabla \times (\mu_r^{-1} \nabla \times E) - k_0^2(\varepsilon_r - j\sigma/\omega\varepsilon_0)E = 0$$

- Alcator C-Mod
  - Equilibrium 1080320017
  - T_{e0} = 2.0 keV
  - $n_{e0} = 8\ 10^{19}\ m^{-3}$
  - $n_{||} = 2.3$



Off-site inversion of EM problem

DELL T7400 workstation
2 quad-cores 3.0GHz
96 GB ram

Assemble EM problem (**A**,**b**) using COMSOL

A (sparse) * **x** = **b**

Cray XT4
9572 quad-cores 2.3GHz
78 TB ram

Invert the sparse linear system using MUMPS library

x = $\mathbf{A}^{-1}$ (sparse) * **b**

Post-processing of solution in COMSOL and MATLAB

Problem with 25M unknowns has been successfully solved

Conclusions

- Plasma wave simulation based on FEM is under development
  - Straightforward modeling of 3D cold plasma
  - Seamless handling from the vacuum to the core plasma
  - Efficient approach (allows fast solution of larger problems)
  - Accelerate the development of antenna design and wave simulation
- Single toroidal mode analysis
  - Electron Landau damping and 1D FP included by an iterative procedure
  - Possibility of accurately modeling the SOL
  - Large scale plasma simulation are at reach using massive parallel computing
  - 2D simulation is evolving towards high-density multi-pass regime
- Working towards comparisons with experiments
  - 2D Fokker Planck to compare with experiment (driven current/Hard X-ray)
  - Take into account the full width of the antenna launched spectrum