Level set based Topology Optimisation of Convectively Cooled Heat Sinks

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Topology optimisation is a mathematical approach that **optimises material layout** for a given **set of constraints** meeting prescribed set of performance **objectives**.

Concept is started for structural mechanics problems (by Bendsoe & Kikuchi) but now it finds application in Fluids, Acoustics, Electromagnetics, Optics etc.

There are different methods for Topology optimisation they are,

- Density Method
- Level set methods
- Topological derivative
- Phase field method and
- Evolutionary approaches.
**Introduction to Topology Optimisation**

**Objective:**
Maximum stiffness or minimum compliance \([F_{TU}]\)

**Constraints:**
30% of material volume
\(KU = F\) (Governing equation)

Ref: O. Sigmund, A 99 line Topology optimisation code written in Matlab, Struc & Multidisc Optim 2001
TO with the Level–set method

- Level set method is a concept developed for studying moving boundaries
- Major steps in Level-set TO
  - LSF parametrization (Polynomial shape function or Radial Basis function)
  - Mapping of geometry into mechanical model, Ersatz material, XFEM, Conforming mesh
  - Optimization strategy (Hamilton Jacobi solver or Mathematical programming)

\[
\psi = \begin{cases} 
0 & \forall x \in \partial \Omega \ (\text{boundary}) \\
> 0 & \forall x \in \Omega^+ \ (\text{solid region}) \\
< 0 & \forall x \in \Omega^- \ (\text{void region})
\end{cases}
\]

Hamilton-Jacobi equation

\[
\frac{\partial \psi}{\partial t} = -v |\nabla \psi| - \omega g
\]

Velocity of propagation

Nucleation of new holes

Design domain

Material

\[ \Omega \]

\[ \partial \Omega \]

\[ \psi = 0 \]

\[ \psi < 0 \]

\[ \psi > 0 \]
TO with the Level–set method

- **Advantages**
  - Accurate prediction of interphases
  - No pressure diffusion in fluid flow problems (in XFEM & Conformal Mapping)
- Compared to Density method, convergence of Level-set method is slow
Level Set TO - Numerical Implementation

1. Initialise Level Set
2. Import Level Set distribution
3. Solve for the Multi-physics problem through FEM
4. Evaluate sensitivity, Objective, Area difference

**COMSOL Multiphysics**

5. Extract the sensitivity, Objective & Area difference
6. Evolve the Level Set (HJ equation)
7. Shape converged?
   - Yes: End
   - No: Re-initialise & Export the updated Level Set

**Matlab LiveLink**
Re-initialisation of Level sets

Eikonal Equation
\[ \frac{\partial \psi}{\partial t} + w \cdot \nabla \psi = S(\psi_0) \]
\[ w = S(\psi_0) \frac{\nabla \psi}{|\nabla \psi|} \]
where S is smoothed sign function

Level Set after few TO iterations, before Re-initialisation

Level Set after Re-initialisation
Heat sink Design: Problem Formulation

Objective: Thermal Compliance: \( \min \int_{\Omega} k_{gam} \ast [(\nabla T)^2] d\Omega \)

Governing Eqns:
\[
\rho C_p (u \cdot T) = \nabla \cdot (k \nabla T) + Q \\
(\nabla \cdot \mathbf{u}) = 0 \\
(u \cdot u) = -\nabla p + \mu \nabla \cdot (\nabla \mathbf{u}) - \alpha u \\
H(\Psi)u = 0
\]

Viscous Dissipation: \( \min \mu \int_{\Omega} \left(\frac{\partial u_i}{\partial x_j}\right)^2 d\Omega \)

Material: \( k_f/k_s = 0.001 \) High conductivity solid
\( k_f/k_s = 0.1 \) Low conductivity solid

Heat flux = 700 W/m²
Reynolds number = 600
Constraints: 40% volume constraint for solid material

<table>
<thead>
<tr>
<th>Variable</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kgam</td>
<td>((K_s - K_f) \ast H + K_f)</td>
</tr>
<tr>
<td>Cpgam</td>
<td>((Cp_s - Cp_f) \ast H + Cp_f)</td>
</tr>
<tr>
<td>ρgam</td>
<td>((\rho_s - \rho_f) \ast H + \rho_f)</td>
</tr>
<tr>
<td>α</td>
<td>((\alpha_{\text{max}} - \alpha_{\text{min}}) \ast H + \alpha_{\text{min}})</td>
</tr>
</tbody>
</table>
High conductivity solid - Results

- Heat sink has tree like/Dendritic shape
- Temperature is uniformly distributed throughout the design domain

Iteration = 67
Area Difference = 2.4384e-05
Thermal Compliance = 202.51
Max Temperature = 523.10K
Low conductivity solid - Results

- Secondary branches have disappeared for low conductivity solid

Iteration = 83
Area Difference = 6.5897e-07
Thermal Compliance = 3154.40
MaxTemperature = 631.59 K
Minimum Viscous Dissipation - Results

- Viscous Dissipation objective leads to a shape guiding the flow with least resistance
Combined objective, tries to minimize both Thermal Compliance & Viscous Dissipation.
Three dimensional Heat sink design

Computational Domain

- Design domain of size 0.1x0.1x0.1m is discretised with 43x43x43 hexahedral cells
- Material: $K_f/K_s = 0.001$ (High conductivity solid)
- Heat flux = 1000W/m$^2$
- $Re = 8$ (vel = 4e-5m/s)

**Objective**: Minimizing the thermal compliance

**Constraints**: 25% volume constraint for solid material
Three Dimensional High conductivity solid - Results

- Tree like structure with primary branches starting from heat source reaching to corners of the domain.
- Use of symmetry condition & Global optimality of the shape needs to be verified.

Therm Compliance=8.2257
Max Temp= 412.519 K
Conclusions

- Implemented Level set based Topology optimisation methodology with Re-initialisation in Comsol 5.2 using MATLAB Livelink® feature

- Demonstrated the application of this methodology for Heat sink designs for different objectives

- Heat sink for thermal compliance objective leads to Dendritic shape whereas for Viscous dissipation objective leads to solid shape guiding the flow with least resistance

- Three Dimensional Heat sink also designed for minimum Thermal Compliance Objective.

- Further research is needed to ensure the global optimality of the obtained shapes
Thank You

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Additional slides
## Results Comparison

<table>
<thead>
<tr>
<th>Re</th>
<th>Pr of fluid</th>
<th>Kf/Ks</th>
<th>Heatflux</th>
<th>Coupled Level Set</th>
<th>SIMP</th>
<th>Re-initialized LS</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>105</td>
<td>0.001</td>
<td>700</td>
<td>29.5 T=509K</td>
<td>43.7 (g=0.2) T=510K</td>
<td>202.506 *</td>
</tr>
<tr>
<td>60</td>
<td>105</td>
<td>0.1</td>
<td>700</td>
<td>2687 T=606K</td>
<td>2569 (g=0.7) Temp:618K</td>
<td>3154.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>SIMP TO</th>
<th>Coupled LS</th>
<th>Re-initialised LS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal compliance (kgm²K/s³)</td>
<td>6.518</td>
<td>2.05</td>
<td>8.2257</td>
</tr>
<tr>
<td>Maximum Temperature (K)</td>
<td>383.9</td>
<td>378.58</td>
<td>412.52</td>
</tr>
</tbody>
</table>

- Coupled LS results show lower objective value than re-initialised LS due to presence of grey cells.
- CFD study on optimal shapes are required to validate the results.
Results with Symmetry Boundary condition

Results for $K_f/ks = 0.1$, $Re = 600$

- $Lm_area = 0.00598$ (-0.3%)
- $Thermcomp = 1400.10$
- Max $T = 621.44k$
- Limiter = 950,000
Low Reynolds number Duct flow

- Initial Values: if(H<1,0[m/s], v1) & Additional 'leaked Wall' condn with No surfaces selected. (Leaked wall condn not necessary; *Lsnoleakwall.mph)
  - Setting this initial value imposes noslip on solid walls
  - Also corrected the force (alpha) term
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Comsol Multiphysics

Matlab Livelink

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