Getting State-Space Models from FEM Simulations

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Eindhoven
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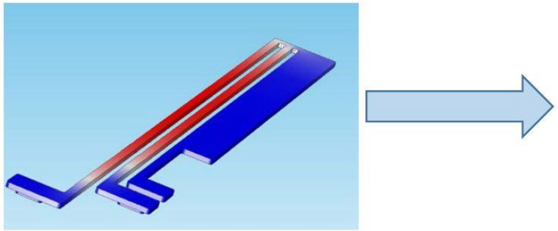
Overview

- **Background on my work**
- **State-Space models, WHAT and WHY**
- **How to get them from FEM**



$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

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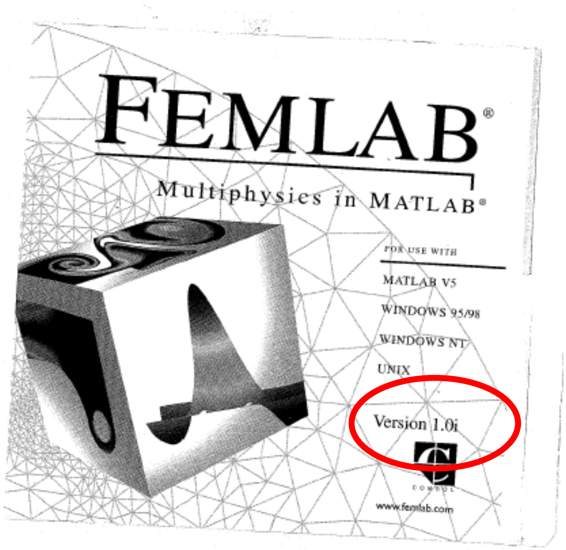
Background

Technische Universiteit Eindhoven University of Technology

Assistant professor

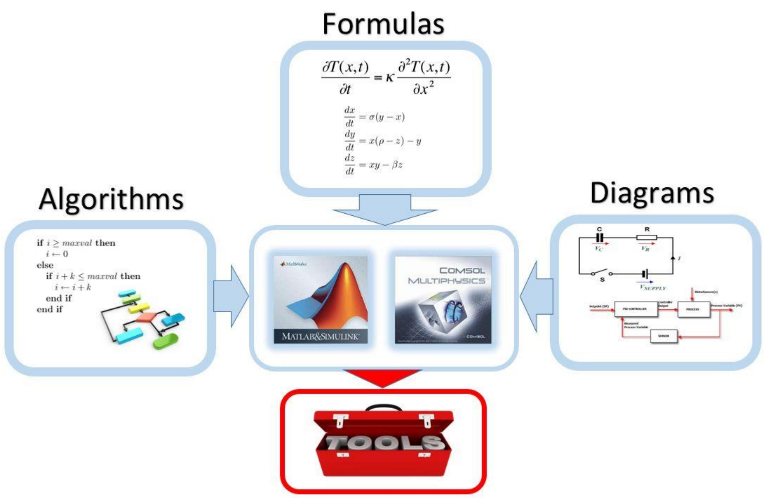
Since 1998 COMSOL® User



CompuToolAble

Entrepreneur

Since 2015



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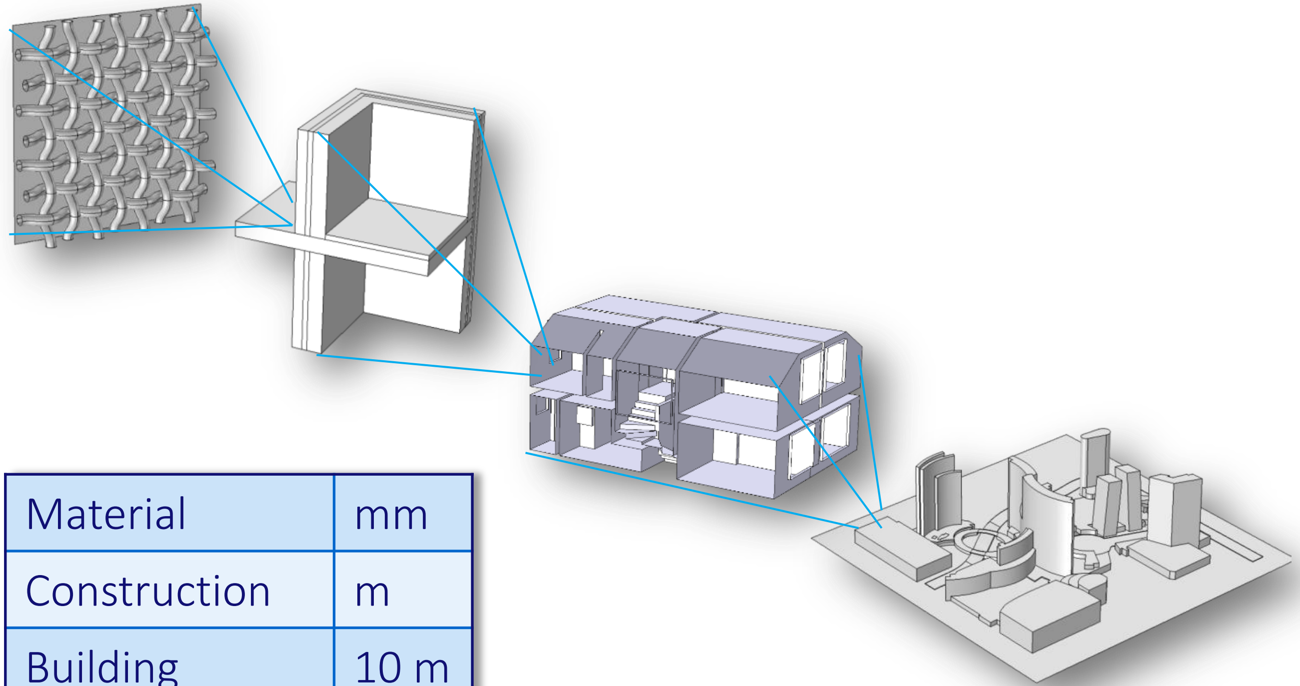
The Built Environment is Multiscale



Material	mm
Construction	m
Building	10 m
Urban Area	1 km

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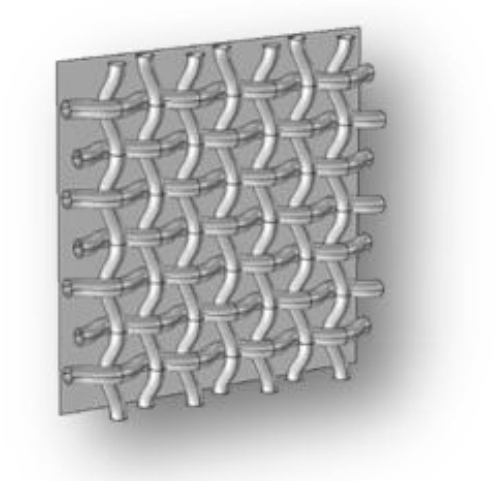
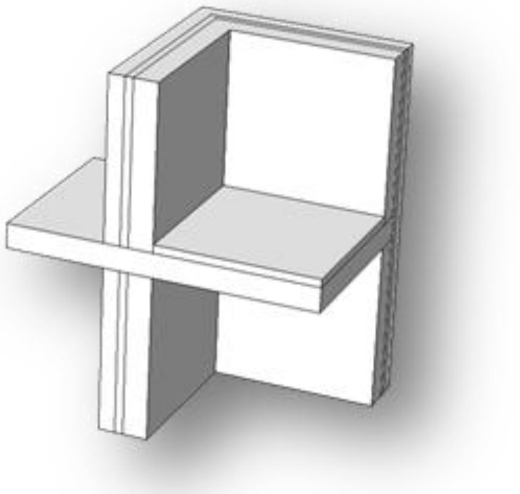
Physics of the Built Environment
Scale level [mm]

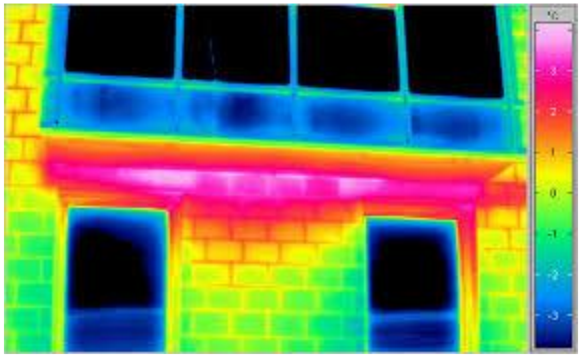


Material ~ mm

Material Physics
- Durability
- Energy

Physics of the Built Environment
Scale level [m]





Construction ~ m

Construction Physics

- Safety
- Durability
- Energy

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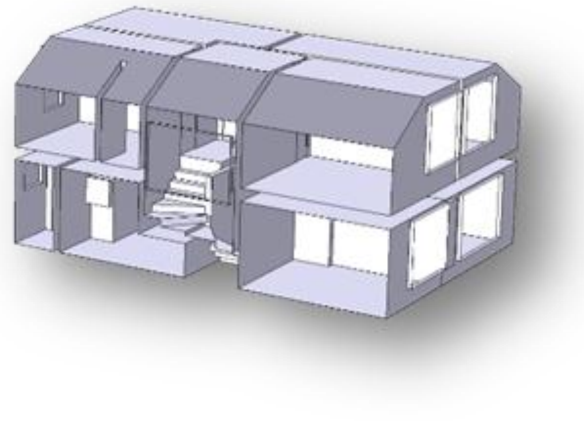
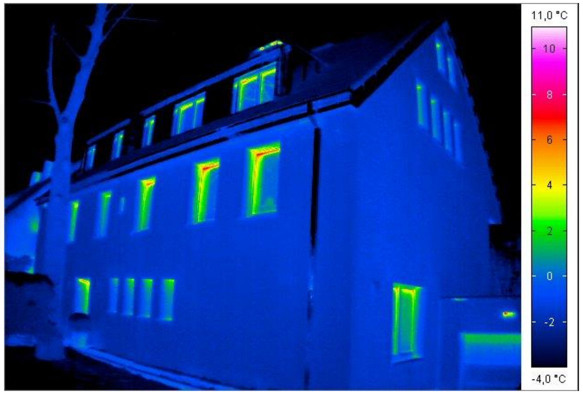
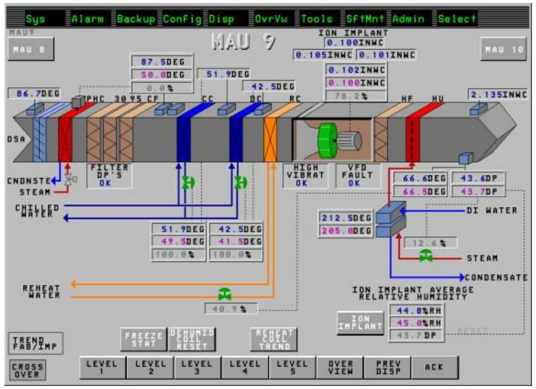
Physics of the Built Environment
Scale level [10 m]







Building ~ 10 m

Building Physics
- Indoor Climate (T,RH,v,Pollutant)
- Building systems
- Health
- Energy

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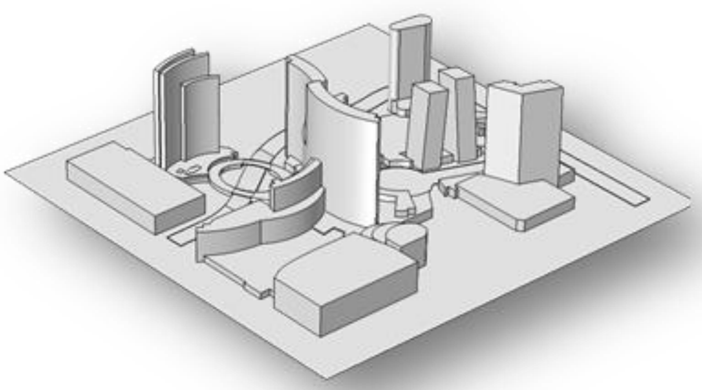
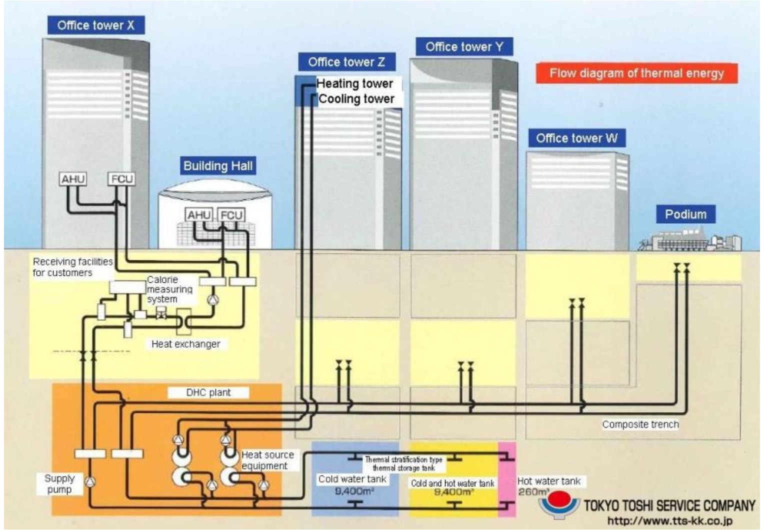
Physics of the Built Environment
Scale level [km]

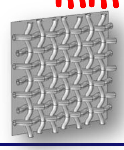
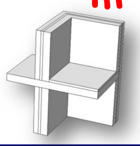
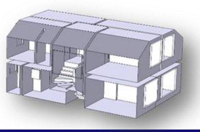
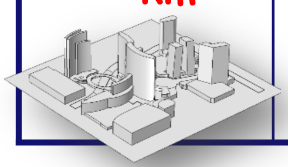




Urban Area ~ km

Urban Physics
- Urban Climate (Pollutant, Wind)
- Urban Systems
- Aquifer
- Energy

Modeling the Built Environment
Physics and Scales

Physics / Scales	Heat	Moisture	Air	Pollutant	Stress
~ mm 					
~ m 					
~ 10m 					
~ km 					

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What are State-Space models?

n x 1 vector — n x n Matrix — n x 1 vector — n x 1 vector

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \vdots \\ \frac{dx_n}{dt} \end{bmatrix} = \begin{bmatrix} \Delta & \cdots & \Delta \\ \vdots & \vdots & \vdots \\ \Delta & \cdots & \Delta \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} \Delta \\ \vdots \\ \Delta \end{bmatrix} u(t)$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u(t)$$

$$y = \mathbf{C}\mathbf{x} + \mathbf{D}u(t)$$

$$y = \begin{bmatrix} \Delta & \cdots & \Delta \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} \Delta \\ \vdots \\ \Delta \end{bmatrix} u(t)$$

1 x n vector — n x 1 vector

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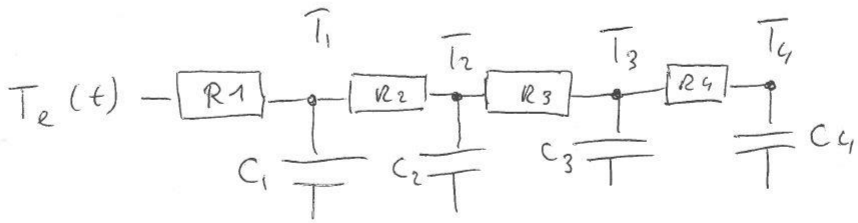
Why are State-Space models so handy?



$$C_1 \frac{dT_1}{dt} = \frac{T_e - T_1}{R_1} - \frac{T_1 - T_2}{R_2}$$

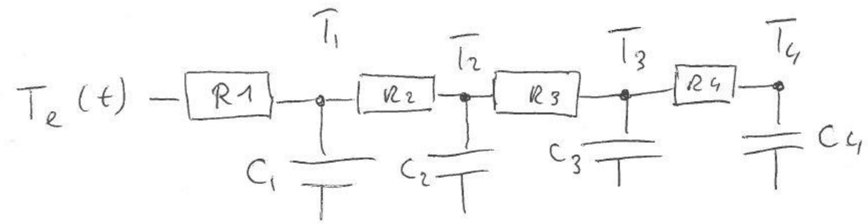
$$C_2 \frac{dT_2}{dt} = \frac{T_1 - T_2}{R_2} - \frac{T_2 - T_3}{R_3}$$

$$C_3 \frac{dT_3}{dt} = \frac{T_2 - T_3}{R_3} - \frac{T_3 - T_4}{R_4}$$

$$C_4 \frac{dT_4}{dt} = \frac{T_3 - T_4}{R_4}$$

```
A(1,1)=(-1/(R1*C1)-1/(R2*C1));
A(1,2)=( 1/(R2*C1));
A(2,1)=(1/(R2*C2));
A(2,2)=(-1/(R2*C2)-1/(R3*C2));
A(2,3)=( 1/(R3*C2));
A(3,2)=(1/(R3*C3));
A(3,3)=(-1/(R3*C3)-1/(R4*C3));
A(3,4)=( 1/(R4*C3));
A(4,3)=(1/(R4*C4));
A(4,4)=(-1/(R4*C4))
%B calc
B(1,1)=(1/(R1*C1));
B(2,1)=0;
B(3,1)=0;
B(4,1)=0;
%C calc
C=[0 0 0 1];
%D calc
D=0;
```

Why are State-Space models so handy?



A=

-8.8889e-04	5.8586e-04	0	0
2.9293e-04	-5.8586e-04	2.9293e-04	0
0	5.8586e-04	-6.7910e-04	9.3240e-05
0	0	0.0501	-0.0501

B=

3.0303e-04
0
0
0

C=[0 0 0 1];

D=0;

Why are State-Space models so handy?

- **Compact notation of dynamic systems**
- **The availability of high quality public domain solvers for state-space systems (Octave, Python, R, …)**
- **These State-Space model solvers are extremely efficient in simulating dynamic responses.**

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State Space systems from FEM

(1) Creating a full state-space matrix system using specific COMSOL functionality and including reduced order systems

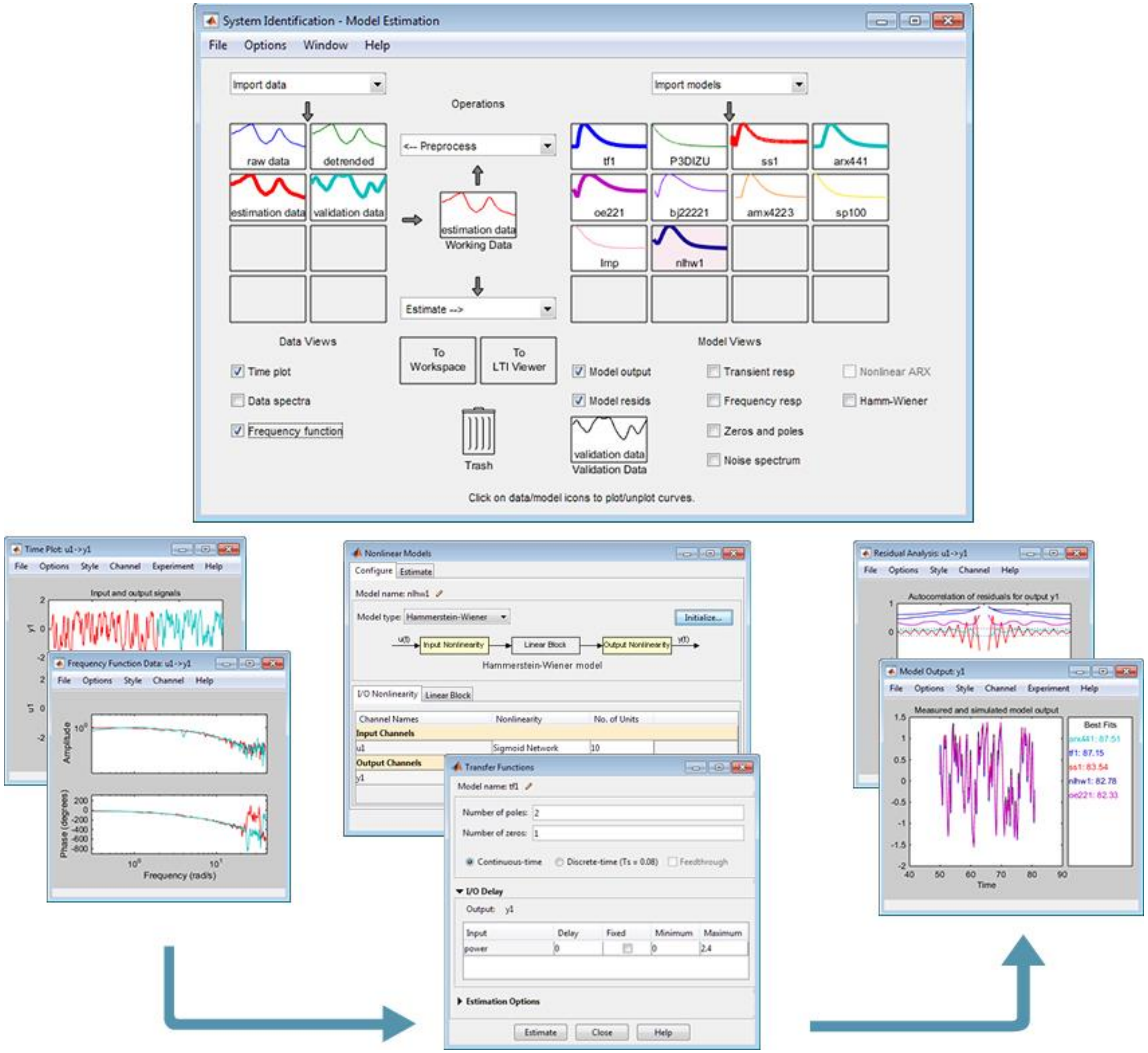
(2) Using identification techniques for example the MatLab identification Toolbox to fit SS systems

(3) Creating a lumped parameter SS model from first principles, where parameters have a physical meaning.

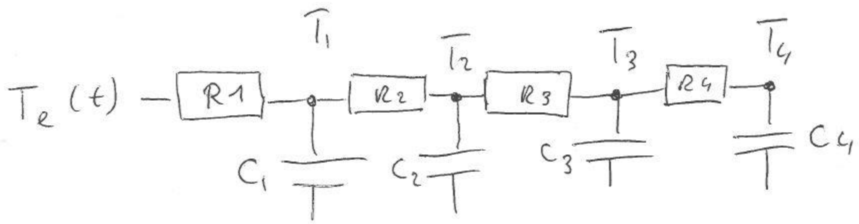
State Space systems from FEM
COMSOL MatLab functionality

```
%Extract full SS model
M2 = mphstate(model,'sol1','out',{'A' 'B' 'C'
'D' 'x0'},...
  'input','mod1.var1', 'output',
'mod1.dom1');


%Create system in MatLab
sys2= ss(M2.A,M2.B,M2.C,M2.D);

%Simulate full SS
y2=lsim(sys2,u,t,M2.x0);

%Reduce order
Options = balredOptions();
sys2Reduced2 = balred(sys2,8,Options);

%Simulate reduced SS
y3=lsim(sys2Reduced2,u,t);
```

State Space systems from FEM
Identification MatLab Toolbox



State Space systems from FEM
First principles



A=

-8.8889e-04	5.8586e-04	0	0
2.9293e-04	-5.8586e-04	2.9293e-04	0
0	5.8586e-04	-6.7910e-04	9.3240e-05
0	0	0.0501	-0.0501

B=

3.0303e-04
0
0
0

C=[0 0 0 1];

D=0;

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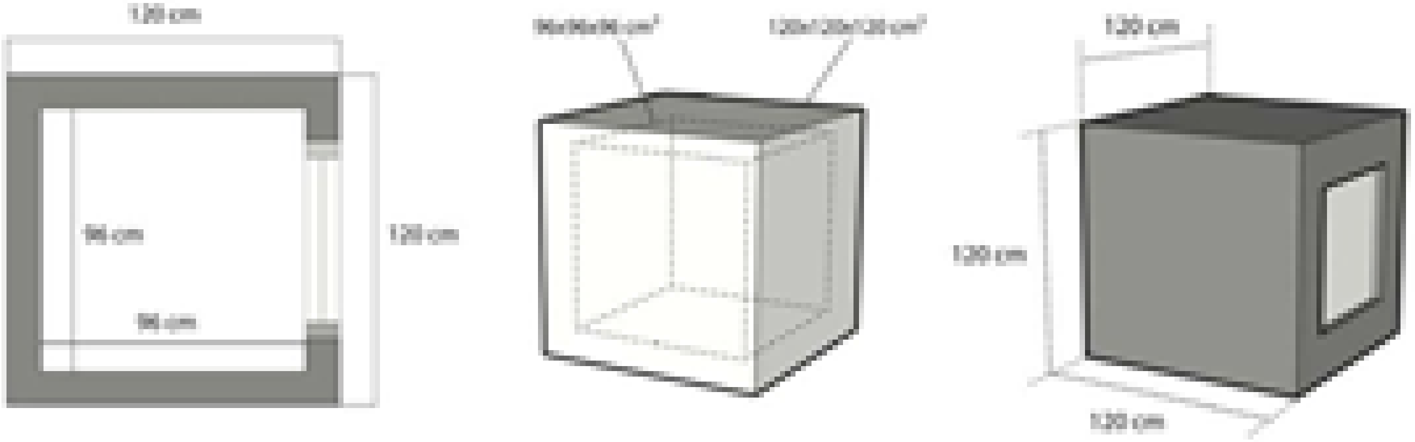
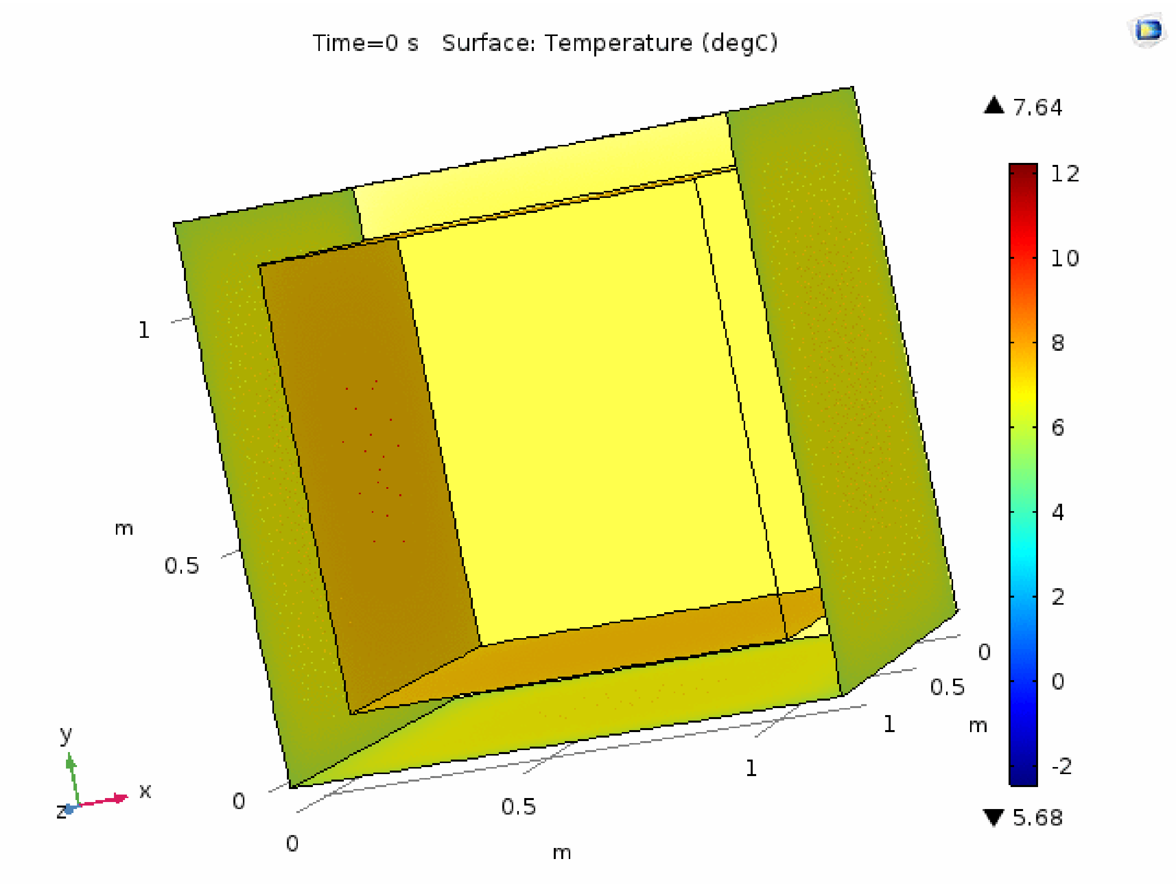
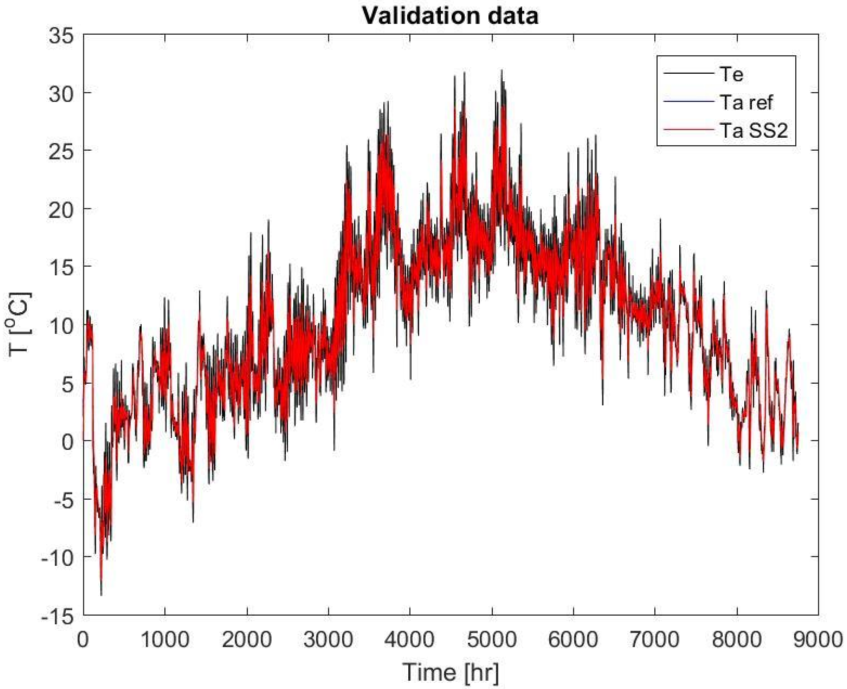
State Space systems from FEM
Results

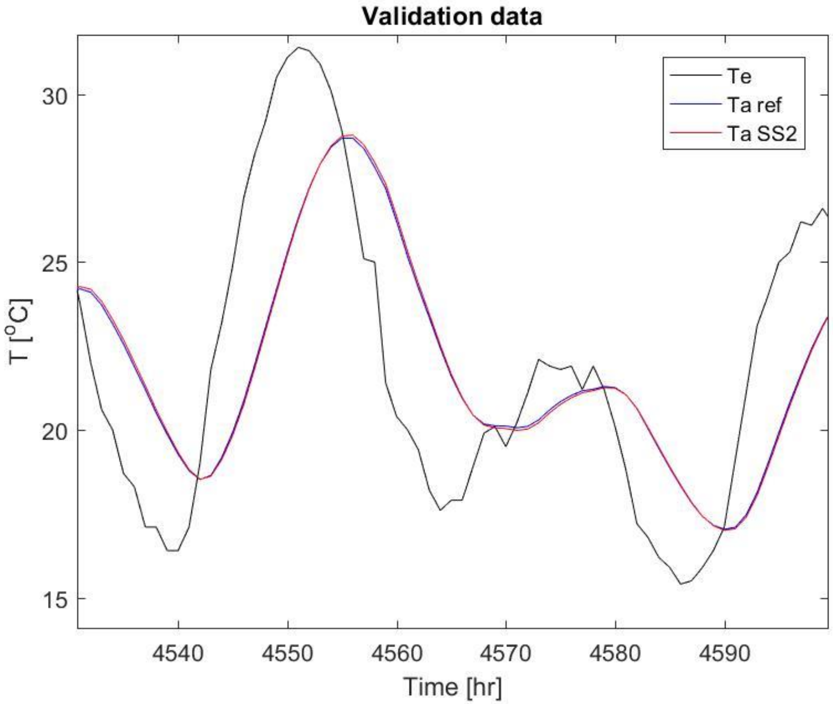


State Space systems from FEM
Results



State Space systems from FEM
Results





State Space systems from FEM
Conclusions

- All three approaches: are capable of significantly reduce computation duration time without loss of accuracy.

- Comparing the three approaches from a physical point of view, the lumped parameter model is preferable
  - because its parameters (state-space matrices) have a physical meaning
  - therefore parameters studies can be done without the necessity to simulate the FEM model over and over again.

- Finally, notice that no general conclusions can be obtained from this rather limited study.