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WIAS

Weierstraß-Institut für Angewandte Analysis und Stochastik

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Numerical Solutions for the Lévêque Problem of Boundary Layer Mass or Heat Flux

Leibniz Gemeinschaft

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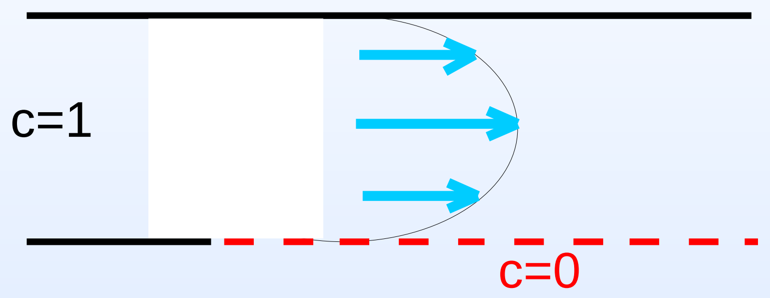
Introduction

- The problem, originally treated by Lévêque in 1928, describes an idealized situation, which appears in many application fields as a limiting case



- There is laminar flow of a free fluid in the gap between two plates of constant spacing *H*.
- Behind an initial undisturbed inflow region one of the boundaries becomes active

Applications

- Heat Transfer
  - Cooling
  - Heating

- Solute Transport
  - Reactive Boundary
  - Catalysis

Mathematical Formulation (Flow)

- Hagen-Poiseuille Flow

$$v(y) = v_{\max} \frac{4}{H} y \left(1 - \frac{y}{H} \right)$$

with maximum velocity

$$v_{\max} = -\frac{H^2}{8\eta} \frac{\partial p}{\partial x}$$

is the analytical solution of the steady state Navier-Stokes equations

for laminar flow between two plates

Mathematical Formulation (Transport)

- Transport (Advection-Diffusion) Equation

$$\nabla D \nabla c = \mathbf{v} \nabla c$$

with diffusivity *D* and velocity *v.*

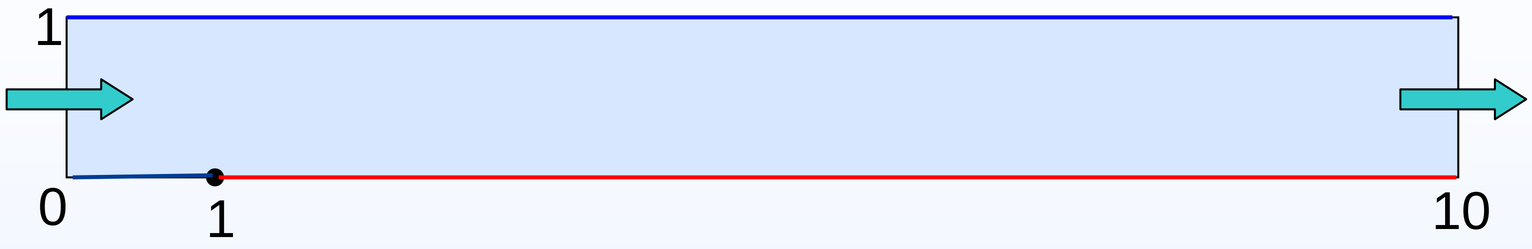
Nondimensionalization yields:

$$\nabla \nabla c = 4y\left(1 - y\right) \mathrm{Pe} \nabla c$$

with Peclet-number Pe.

- Boundary Condition
  - kinetic: $$D \frac{\partial c}{\partial n} = kc$$
  - Infinitely fast: $$c = 0$$

Model Region and Boundaries



Closed reactive boundary

Closed non-reactive boundary

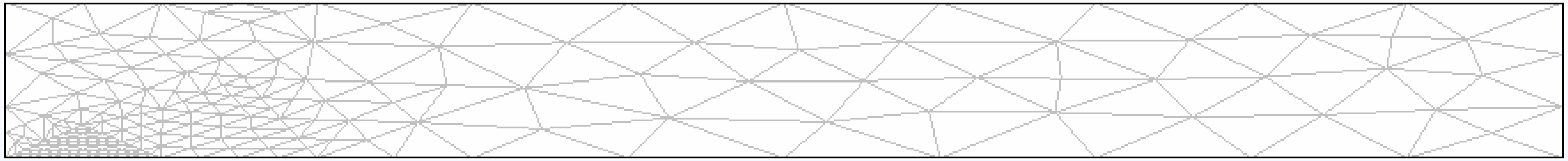
Inflow, Outflow

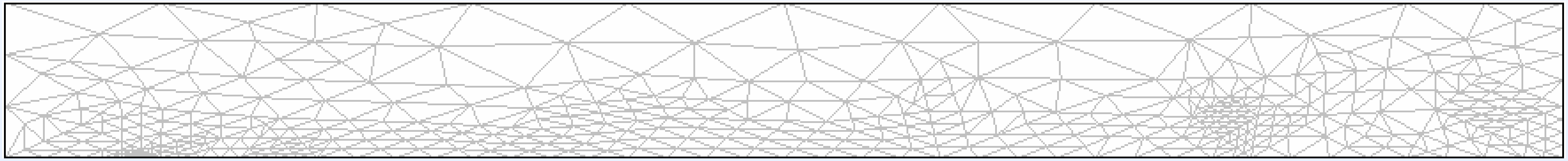
specified concentration at inflow (Dirichlet condition)
convective flux at outflow (Neumann condition)

$$\frac{\partial c}{\partial x} = 0$$

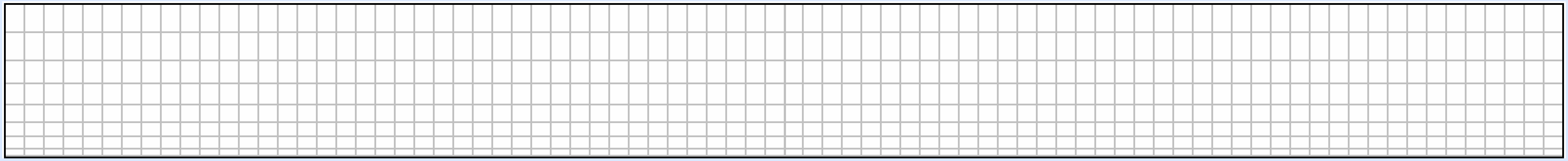
Meshing

- Free Meshes, adaptive grid refinement





- Mapped Meshes
  - Equidistant mesh in horizontal direction (up to 800 nodes)
  - Grid refinement near reactive boundary (up to 100 nodes)



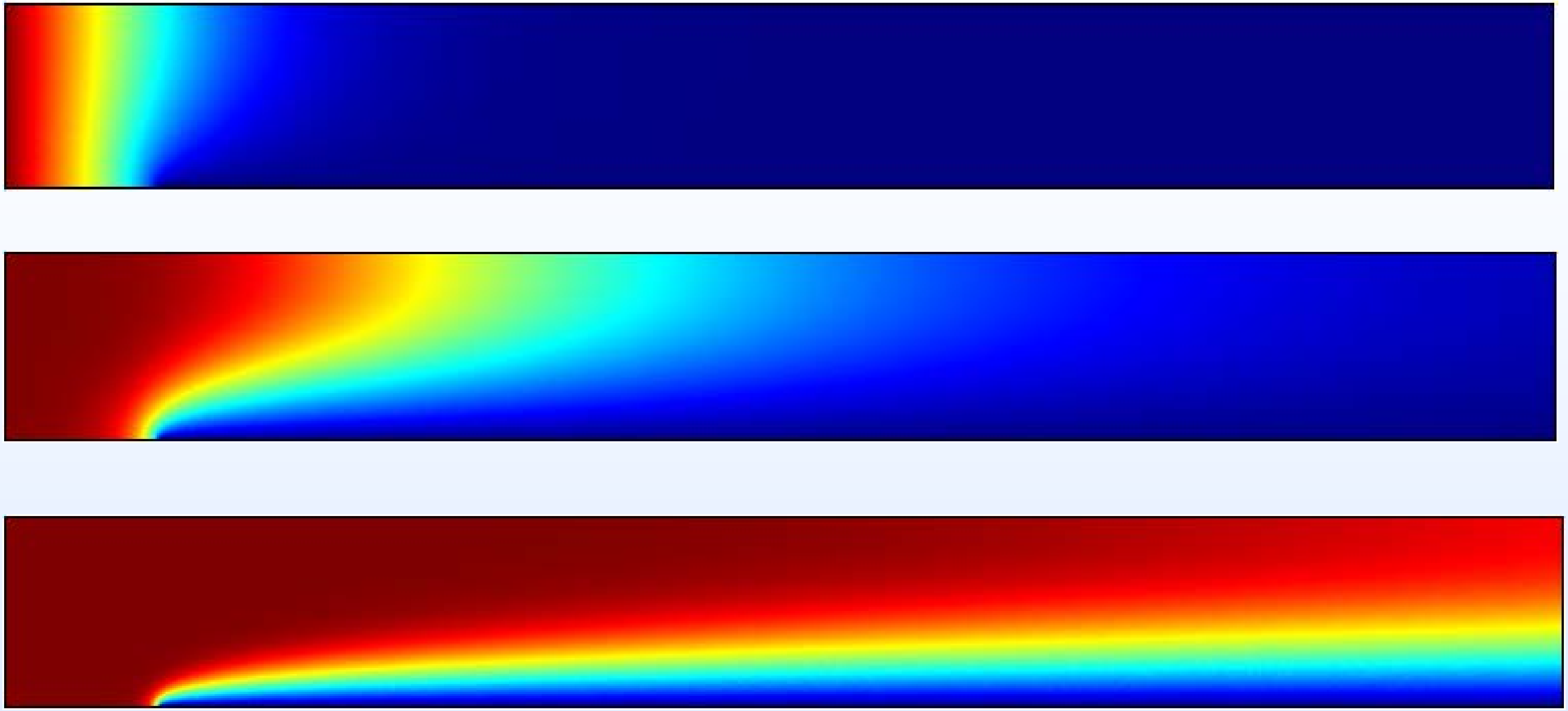
Refined meshes in dependence of the Peclet number

Pe	DOF after 1. refinement	DOF after 2. refinement
0.1	46720	125326
1	46693	122305
10	43837	114268
100	37924	99202
1000	36856	101974
10^4	40951	110032
10^5	43177	123433
10^6	43231	127339
10^7	42613	120868
10^8	42574	124858

Adaptive refinement of free meshes

DOF = degree of freedom

Results for Pe=0.1, 1 and 10

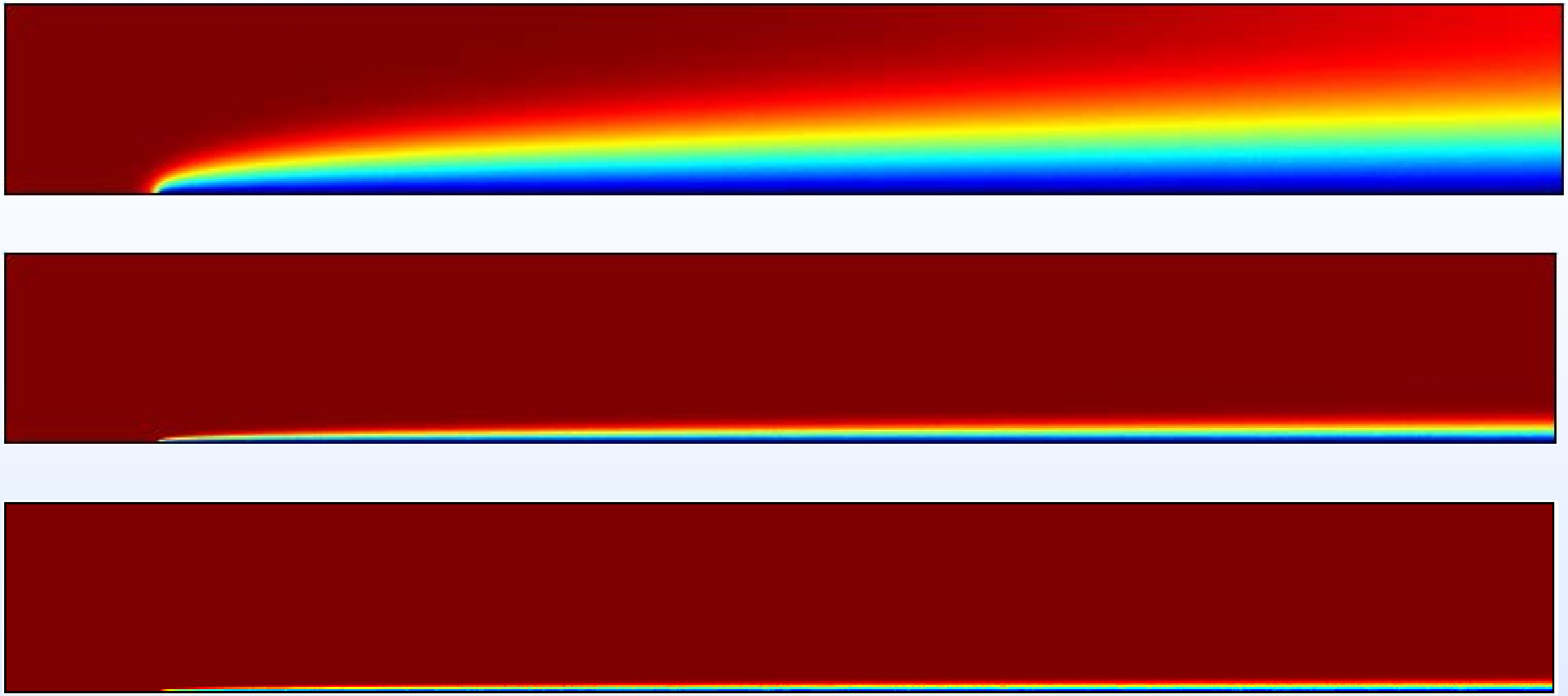


Concentration distribution:

Red: inflow concentration/ temperature

Blue: boundary concentration/ temperature

Results for Pe = 10, 100, 1000



Concentration distribution

With increasing Peclet number:

Shrinking of the reactive boundary layer

Total Heat or Mass Transfer

- Total heat transfer is given by:

$$\mathrm{Nu} = \frac{1}{L}\int \frac{\partial T}{\partial y}\, dx$$

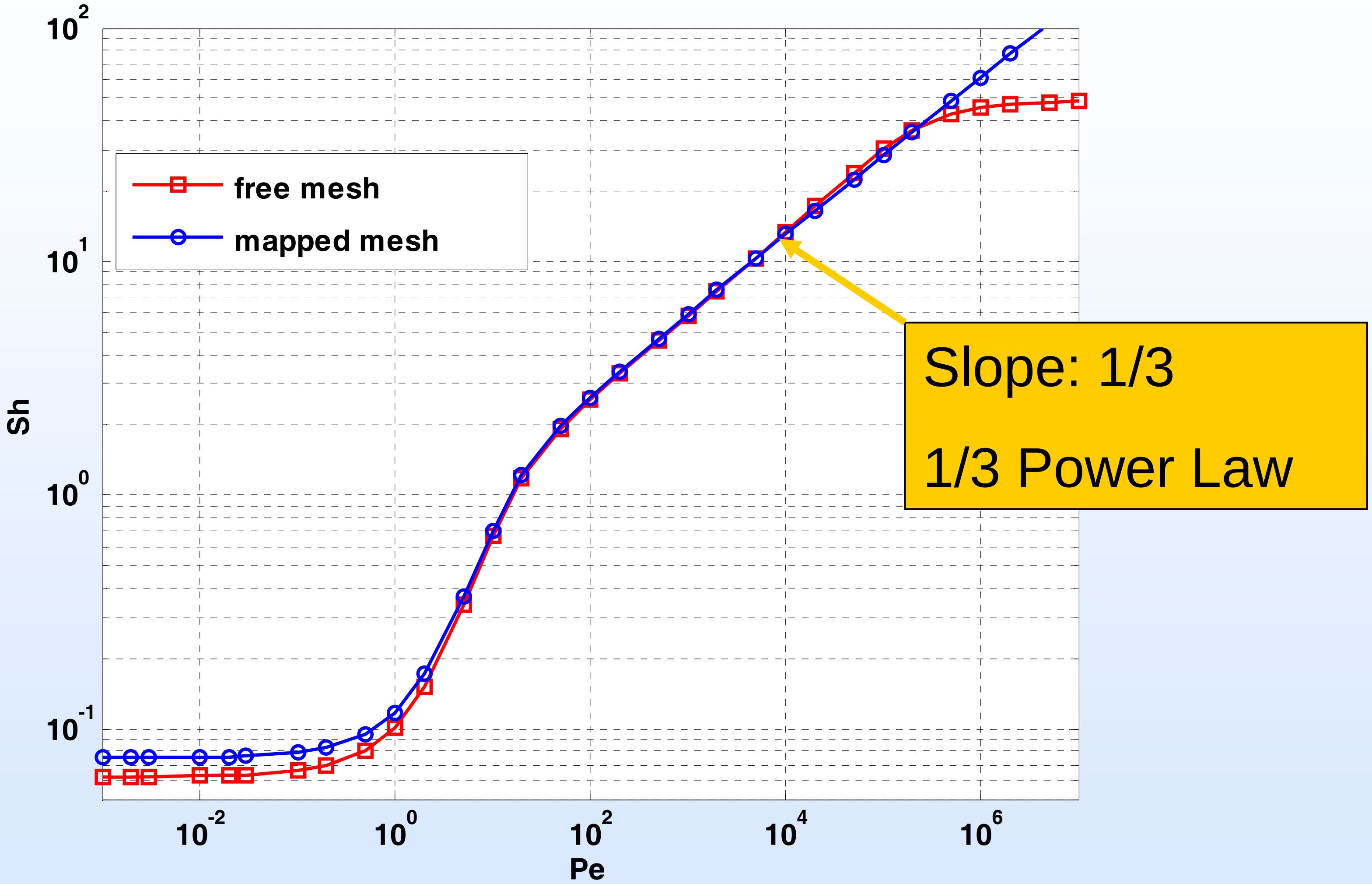
the (dimensionless) Nusselt number Nu

- Analogously for total mass transfer holds:

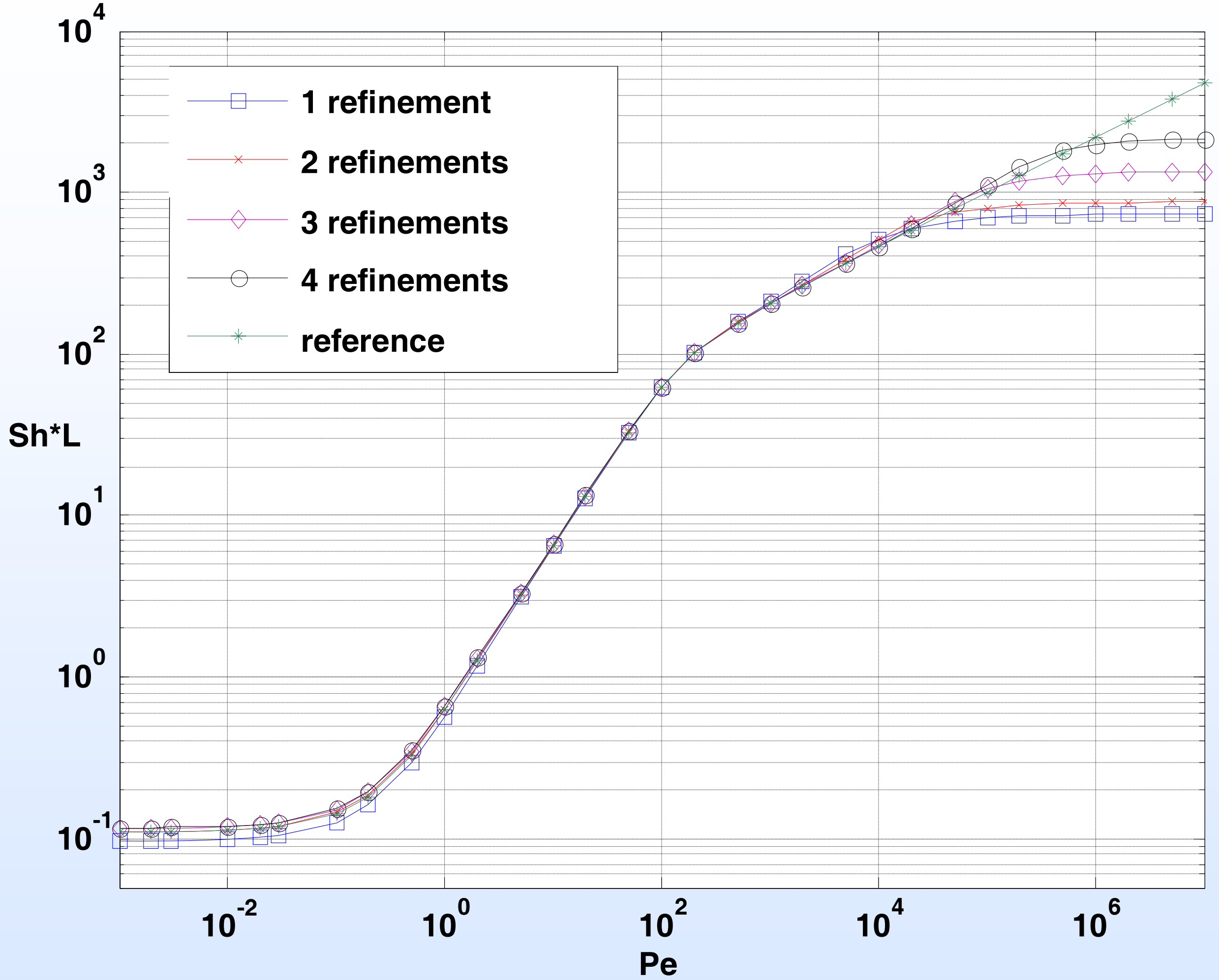
$$\mathrm{Sh} = \frac{1}{L}\int \frac{\partial c}{\partial y}\, dx$$

the (dimensionless) Sherwood number Sh

Sherwood Number Intercomparison 1

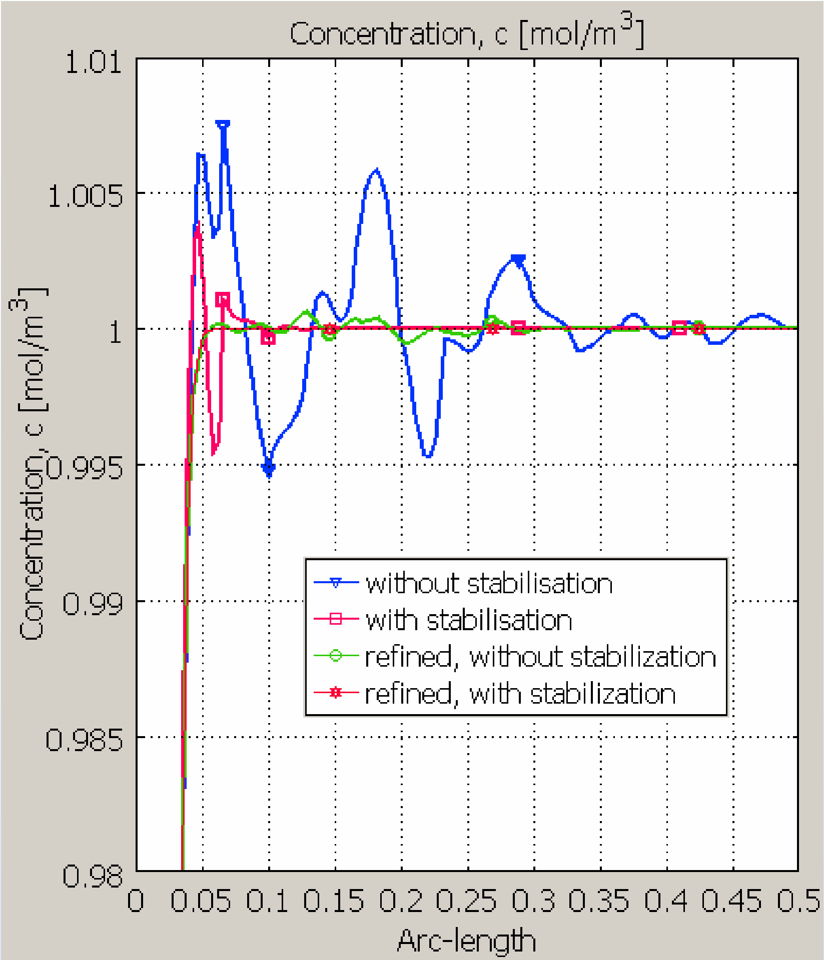


Sherwood Number Intercomparison 2



Refinements for free meshes,
Mapped mesh result is the reference

Effect of Stabilization



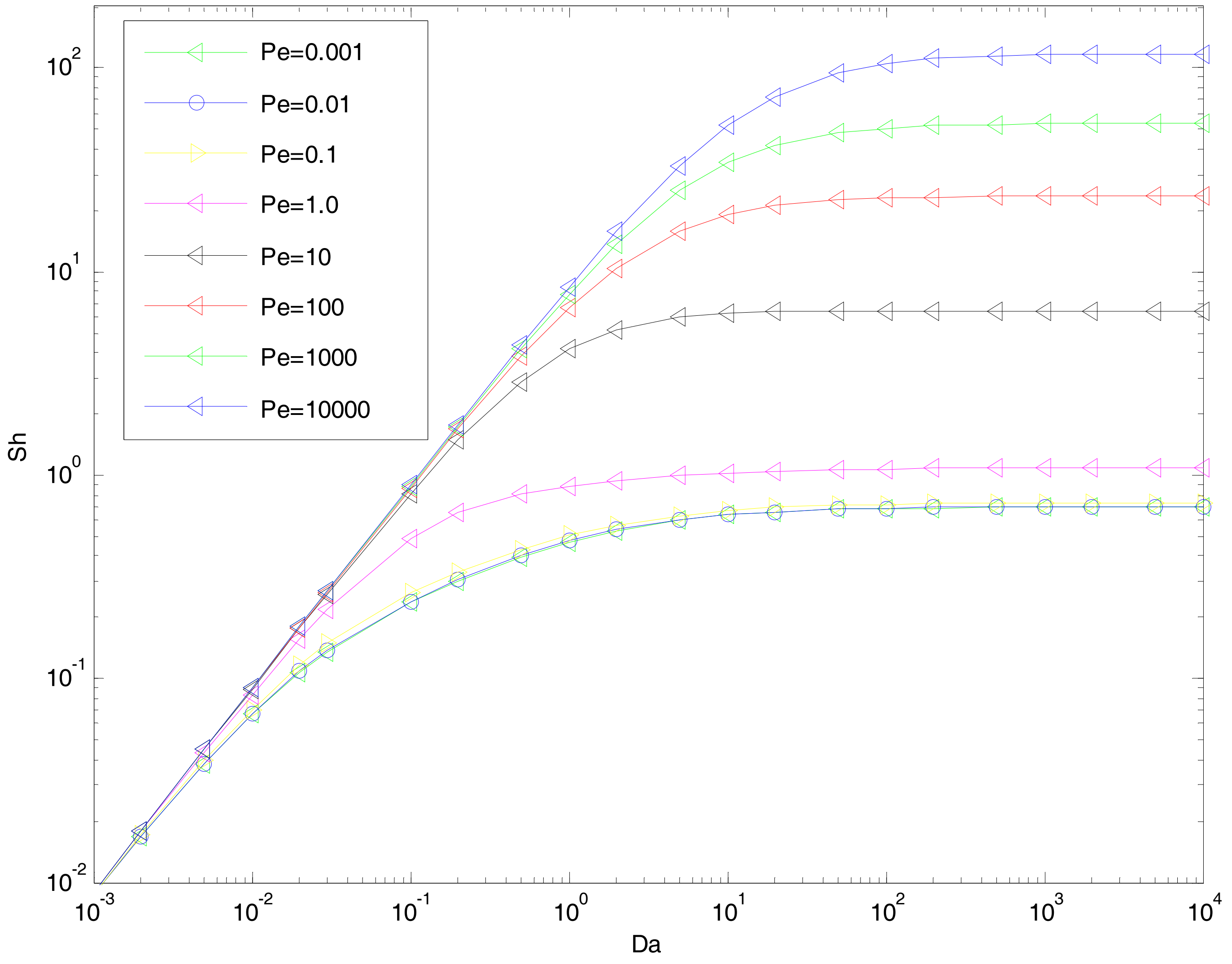
Example runs:

Pe=10^6

Anisotropic streamline diffusion with parameter 0.1

Concentration profile at the lower part of the outlet boundary

Kinetic Reaction



Damköhler

number

Da:=k/D

Conclusions

- The Lévêque 1/3 power law is perfectly confirmed by the numerical results
- The transition between the two asymptotics appears for Péclet numbers between 0.3 and 30
- The mentioned transition regime is already captured accurately by coarse mesh simulations
- Mapped mesh simulations provide more accurate results than free mesh simulations
- For numerical methods it is a higher challenge to approximate the asymptotic situations than the transition regime

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