Modeling of Radial Source Flow in Porous Media: Miscible Viscous Fingering Patterns

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Abstract

Introduction: Viscous fingering, a hydrodynamic instability characterized by complex patterns, is observed when a less viscous fluid displaces a more viscous one in porous medium [1]. The displacement is either rectilinear or radial; both having equal importance in many real life applications. The fingering patterns in radial Hele-Shaw flows have been studied both experimentally [2], and numerically [3]. Various flow parameters and different engravings on the Hele-Shaw plates change the fingering dynamics significantly [2]. Li et al. presented a numerical technique to characterize the formation and dynamics of complex interface morphologies due to instabilities [3]. Although studied extensively, understanding radial fingering dynamics fascinates us due to its analogy to many other phenomena, such as diffusion limited aggregation [4], crystal growth [5], the growth of bacterial colonies and snowflakes [6], etc. Here, we present the simulations of the radial source flow to manifest fingering patterns and their dependence on the log-mobility ratio, volumetric flow rate, and the diffusion coefficient. We have successfully captured that the instability enhances with log-mobility ratio and flow rate.

Use of COMSOL Multiphysics® software: Following Pramanik and Mishra [7], we use the Two-Phase Darcy's Law (tpdl) model of CFD Module of COMSOL Multiphysics® software. The domain of simulations is an annulus of inner and outer radii 2 mm and 58 mm, respectively. The annulus is initially filled with a fluid (fluid1) of viscosity μ1 , which is displaced radially with uniform velocity U0 from the inner circular region by another fluid (fluid2) of viscosity μ2 (<μ1). The inlet boundary condition specifies normal inflow velocity, U0, and saturation of fluid1, s1 = 0. The outlet boundary condition is p = 0, which corresponds to free flow across this boundary. The initial condition is s1 = 1 and p = 0. User-controlled extra fine `free triangular mesh' consisting of 16692 domain elements and 272 boundary elements is used.

Results: Figure 1 shows the spatio-temporal evolution of s1 for R = 2; U0 = 0.002 m/s, where R = ln(μ2 / μ1) is the log-mobility ratio. It is evident that radial flow results in fan/flower-like patterns as mentioned by Miranda and Widom. [8]. Further, it is observed that the tips of the outward (inward) fingers at different times lie on concentric circles. As the time progresses, the fingers get wider and their length increases, while the number of fingers remain the same. Changing R, these qualitative features remain almost unaltered.

Conclusion: With the help of the simulations built with COMSOL Multiphysics® software, we successfully capture miscible VF in radial displacement and its dependence on various flow parameters. It is observed that instability varies significantly with these parameters and hence we can say that the instability can be controlled by wisely choosing the flow parameters. The analysis of simulated data using MATLAB® (Livelink for MATLAB®) to understand the quantitative features of the dynamics is the focus of our ongoing research.

Reference

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Figures used in the abstract

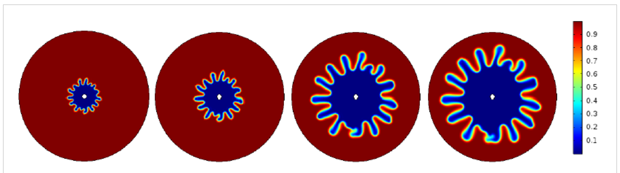

Figure 1: Spatio-temporal evolution of saturation s1 for R = 2, U0 = 0.002 m/s at time t = 10; 20; 60; 80 seconds (from left to right). Here, R = ln(μ2 / μ1) is the log-mobility ratio.