Weak Form & LiveLink for MATLAB Based Modified Uzawa Method for Solving Steady Navier-Stokes Equation

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Introduction: We use a new interactive method (by the script language in LiveLink for MATLAB) to solve the steady incompressible N-S equation (the mixed element form in weak form PDE).

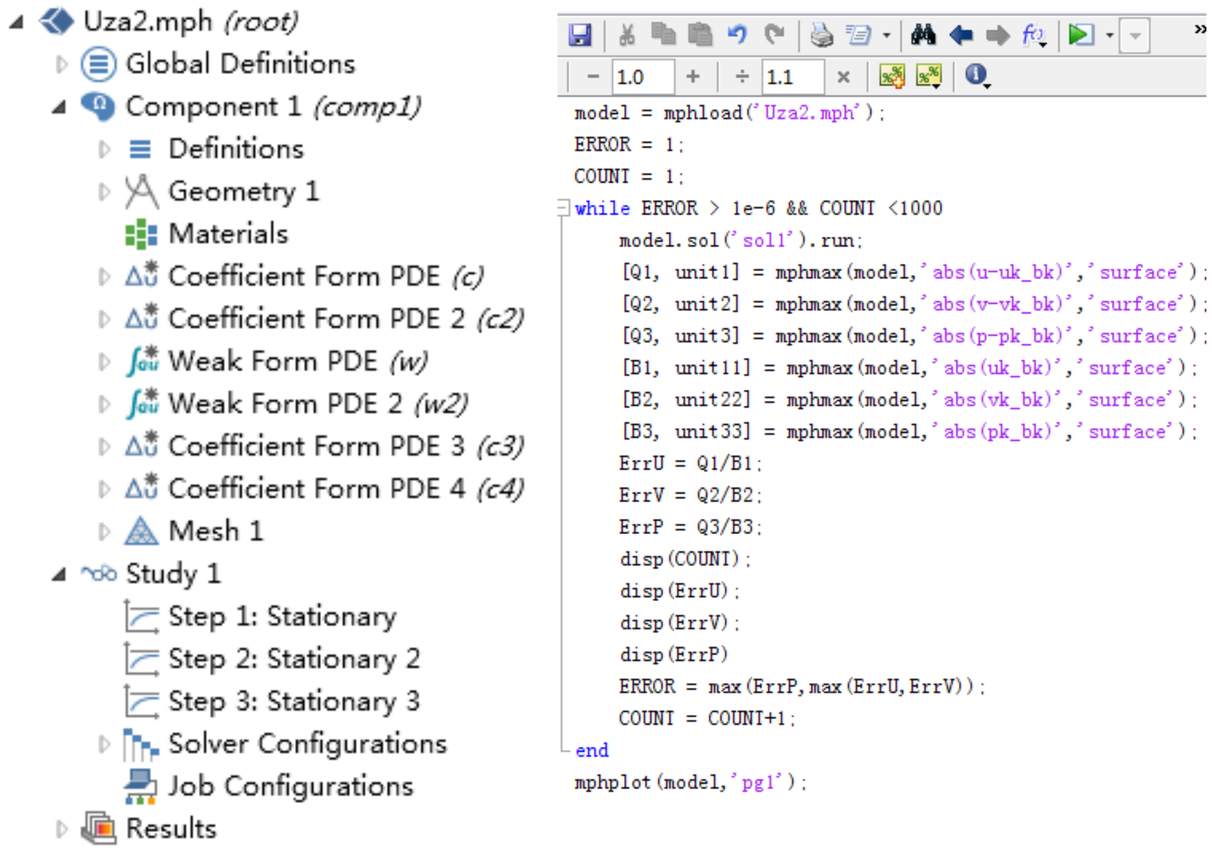

Fig 1. Model Structure and MATLAB Script

Method: In each interactive step of the modified Uzawa method, we solve the decoupled velocity field and pressure field in turn.

NS Equation
$$\begin{cases} -\mu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{f} \\ \nabla\cdot\mathbf{u} = 0 \end{cases}$$

MUM
$$\begin{cases} a_1(\mathbf{u}_h^n;\mathbf{u}_h^{n+1},\mathbf{v}) + \mu(\nabla\mathbf{u}_h^{n+1},\nabla\mathbf{v}) \\ \qquad -(p_h^n,\operatorname{div}\mathbf{v}) = (f,\mathbf{v}) \quad \forall\mathbf{v}\in V_h \\ (p_h^{n+1},q) = (p_h^n,q) - \rho(\operatorname{div}\mathbf{u}_h^{n+1},q) \\ \qquad\qquad \forall p\in P_h \end{cases}$$

Here $a_1(\mathbf{u};\mathbf{v},\mathbf{w}) = \int_\Omega (\mathbf{u}\cdot\nabla)\mathbf{v}\cdot\mathbf{w}\,d\Omega$

Tests: Three tests are given as follows.

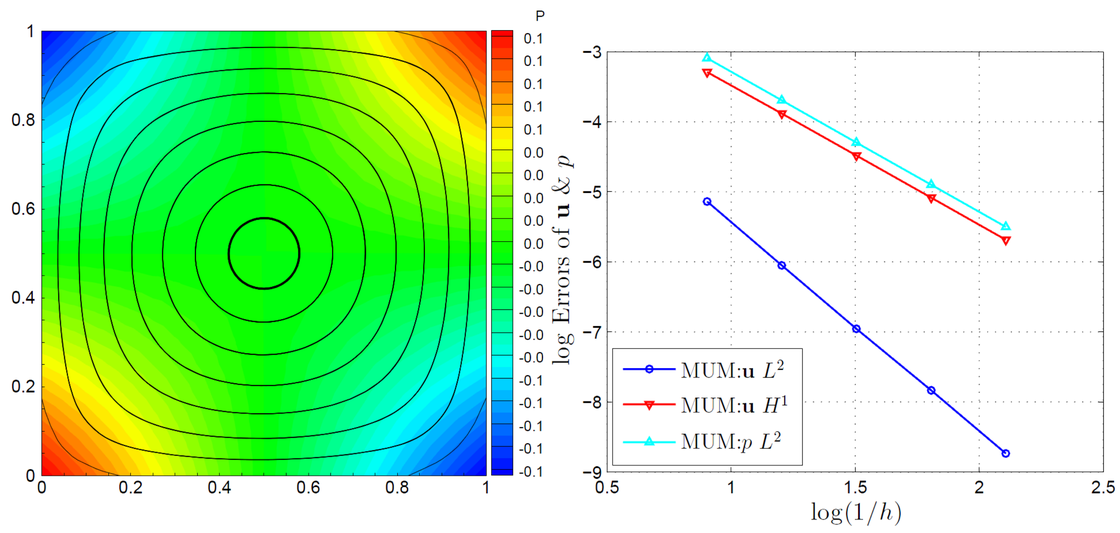

Fig 2. Solution and Err. Analysis of Test 1

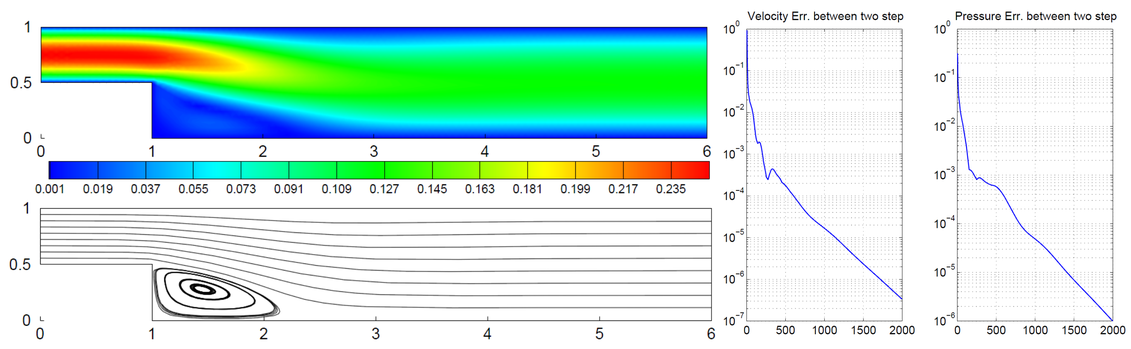

Fig 3. Solution Value and Analysis of Test 2

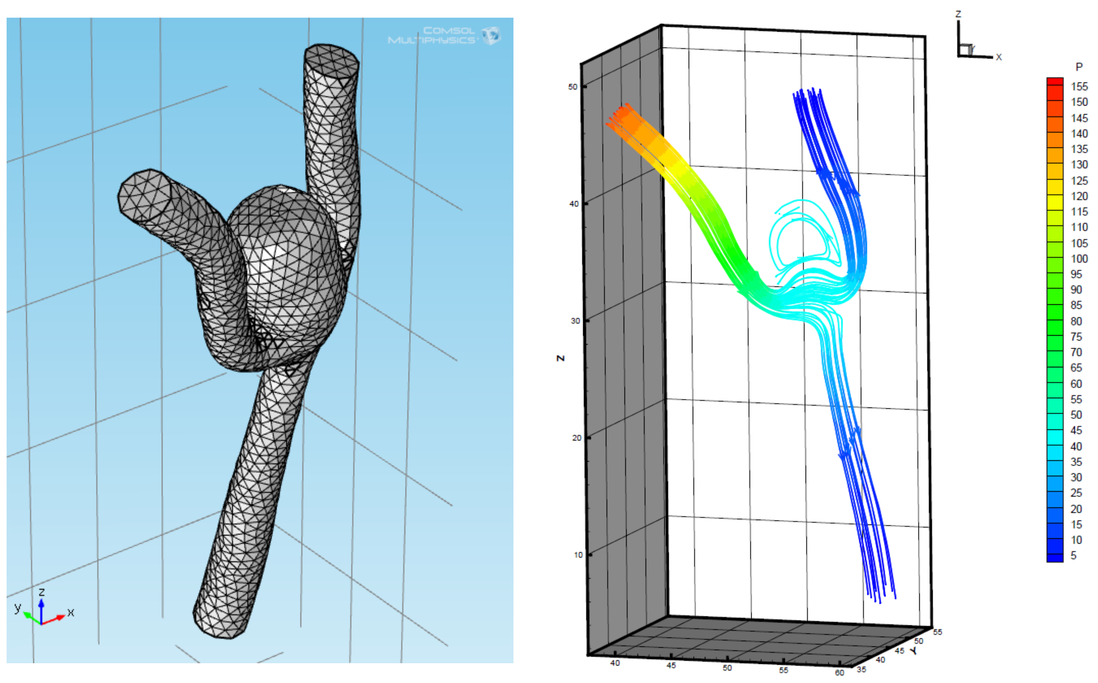

Fig 4. Test 3 : Aneurysm Simulation

Conclusion: The weak form PDE and script coding can solve problems more flexible and more powerful.

Reference:

[1] P. Chen, J. Huang, H. Sheng, Some Uzawa methods for steady incompressible Navier–Stokes equations discretized by mixed element methods, Journal of Computational and Applied Mathematics, 273, 313-325 (2015)