Thermal Model for Single Discharge EDM process

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Outline

Thermal (Anode – Cathode Model)

- Single Discharge: Heat Conduction and Joule Heating
- GUI (Comsol – Matlab)
- Residual Thermal Stress
Physical Model

- Heat Conduction
- Joule Heating
- Input Parameters

(3D Plot for Temperature Profile (Copper – Steel))
Heat Conduction

- **Governing Equation:**
  - Heat Equation: \( \rho C_p \frac{\partial T}{\partial t} + \rho C_p u \cdot \nabla T = \nabla \cdot (k \nabla T) + Q \)

- **Boundary Conditions:**
  - Initial Values
  - General Inward Heat Flux \( -\mathbf{n} \cdot (-k \nabla T) = q_o \)
  - Thermal Insulation \( -\mathbf{n} \cdot (-k \nabla T) = 0 \)
Joule Heating

- Governing Equations:
  - $\mathbf{J} = \left( \sigma + \varepsilon_0 \varepsilon_r \frac{\partial}{\partial t} \right) \mathbf{E} + \mathbf{J}_e$
  - $\nabla \cdot \mathbf{J} = Q_j$
  - $\mathbf{E} = -\nabla V$

- Boundary Conditions:
  - Initial Values
  - Normal Current Density ($-\mathbf{n} \cdot \mathbf{J} = J_n$)
  - Electrical Insulation ($-\mathbf{n} \cdot \mathbf{J} = 0$)
Input Parameters

• Plasma Channel Expansion:
  ▪ Empirical
  ▪ Constant Cathode Boiling Temperature
  ▪ Eubank Physical Model

• Flux Profile:
  ▪ Equilibrium
  ▪ Gaussian
  ▪ Parabolic

• Energy Balance:
  ▪ Constant
  ▪ Eubank Physical Model
  ▪ User Defined
Input Parameters: Plasma Channel Expansion

Empirical equation

\[ r_0 + C \times (t)^n = r_p \]
\[ r_0 + C \times (I)^m \times (t)^n = r_p \]

Constant Cathode boiling temp.

\[ T_b = \frac{E_0 \cdot r_p}{K \cdot \sqrt{\pi}} \tan^{-1} \left[ \frac{4at}{r_p^2} \right] \]

\[ r_p = (2.04e-6) \times (I)^{0.4} \times (t)^{0.7} \]
Input Parameters: Flux Profile

Equilibrium:

\[ q_0 = \frac{power}{\pi \cdot r^2} \]

Gaussian:

\[ q_0 = \left(\frac{power}{\pi \cdot r^2}\right) \cdot \exp\left(-4.5 \cdot \left(\frac{r^2}{R^2}\right)\right) \]
Input Parameters: Flux Profile

Parabolic:

\[ q_0 = \left( \frac{power}{\pi \cdot r^2} \right) \times \left( 1.0002 - \frac{r^2}{R^2} \right) \]
Input Parameters: Energy Balance

- **Single discharge**: Plasma-material interaction
  - Total power $E = \text{voltage} \ (U) \cdot \text{current} \ (I) \cdot \text{pulse duration} \ (t)$

- $E_A, E_C$ : Constant, User Defined or from Eubank’s Model
Results

- Graphite, $150 \, (\mu s)$ (Empirical Equation)
Results

Crater radius and depth (\(\mu m\)) vs time (\(\mu s\))

- Crater Radius
- Crater Depth

**Graph Details:**
- **Y-axis:** Crater Radius/Depth (\(\mu m\))
- **X-axis:** Time (\(\mu s\))

**Legend:**
- Blue line: Crater Radius
- Red line: Crater Depth

Graph shows the increase in crater radius and depth over time.

Date: 4/10/2012
GUI (Comsol – Matlab)
Need for GUI?

- Main use of GUI was to get isotherm plots in matlab figure window for easy editing of plots, obtaining curves for crater radius and crater depth based on results obtained from Comsol.

- Other than that it could update geometrical parameters, change materials and their properties, add time dependent equation for current or time energy balance based on interpolation of values, run the study and get results in Matlab variables by using GUI.
## GUI Window: Materials Tab

### Materials:

#### Material for Electrode:
- **Material for Electrode:**
  - Poco Graphite
  - Graphite (Comsol)
  - Copper (Comsol)

#### Density:
- 8700 [kg/m³]

#### Thermal Conductivity:
- 400 [W/(m·K)]

#### Specific Heat:
- 385 [J/(kg·K)]

#### Relative Permittivity:
- 1

#### Electrical Conductivity:
- 5.998·10⁷ [S/m]

#### Material for Workpiece:
- **Material for Workpiece:** Steel W300

#### Density:
- $f(T=2700[K], 0.000199 [kg/m^3], (T/1[K])^2, 0.5597 [kg/m^3], (T/1[K])^2)$
- $3.333·10^{-8} [W/(m^3·K)]·(T/1[K])^3, 0.779·10^{-8} [W/(m^3·K)]·(T/1[K])^2, 0.0751$

#### Thermal Conductivity:
- $f(T=3000[K], 0.000644 [J/(kg·K)], (T/1[K])^2, 0.2912 [J/(kg·K)], (T/1[K])^2)$

#### Specific Heat:
- 1

#### Relative Permittivity:
- 1

#### Electrical Conductivity:
- $4.032·10⁷ [S/m]$
GUI Window: Study Tab

Study:

Load model: C:\Users\kusham\Desktop\gui_comsol\hgraphite_steelprop.mph

Time range: range(10e-6, 10e-6, 150e-6)

Save As: C:\Users\kusham\Desktop\gui_comsol\new_file\hgraphite_steelprop.mph

Update Geometry  Run Study
## GUI Window: Results Tab

### Plot Isotherms

<table>
<thead>
<tr>
<th>Temperature for Isotherms:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>T1 (degC):</strong></td>
</tr>
<tr>
<td><strong>T2 (degC):</strong></td>
</tr>
<tr>
<td><strong>T3 (degC):</strong></td>
</tr>
<tr>
<td><strong>T4 (degC):</strong></td>
</tr>
<tr>
<td><strong>T5 (degC):</strong></td>
</tr>
</tbody>
</table>

### Plot Results

<table>
<thead>
<tr>
<th>Temperature (degC): 700</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Plot Temp(work) Radius</strong></td>
</tr>
<tr>
<td><strong>Plot Temp(elec) Radius</strong></td>
</tr>
<tr>
<td><strong>Plot Temp(work) Depth</strong></td>
</tr>
<tr>
<td><strong>Plot Temp(elec) Depth</strong></td>
</tr>
</tbody>
</table>

- **Generate Plots**
- **Animation**
Residual Stress Modeling
(Using Thermal Stress Module)
Residual Stress

1. Physics
   - Governing equations
   - Boundary Conditions

2. Results
   - Residual Stress Isocontours
   - Residual Stress with Depth along central Axis

3. Further work
   - Rapid cooling effect
Physics: Governing Equations

- \(-\nabla \cdot \sigma = F_v\), \((\sigma = s)\)

- \((s - S_o = C: (\epsilon - \epsilon_o - \epsilon_{inel})), (\epsilon_{inel} = \alpha(T - T_{ref}))\)

- \(\epsilon = \frac{1}{2}((\nabla u)^T + \nabla u)\)

- \(\rho C_p \frac{\partial T}{\partial t} + \rho C_p u.\nabla T = \nabla.(k \nabla T) + Q\)
Physics: **Boundary Conditions**

- **Initial Values**
- **Thermal Insulation** \((-\mathbf{n} \cdot (-k \nabla T) = 0\)\)
- **Heat Flux** \((-\mathbf{n} \cdot (-k \nabla T) = q_o\)\)
- **Fixed Constraint** \((\mathbf{u} = 0)\)
Results: Residual Stress Isocontours
Results: Residual Stress with Depth
Thank You for your attention!

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