Shape and Topology Optimization of Loudspeaker Drivers

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Abstract: This paper illustrates the use of formal mathematical optimization techniques for engineering solutions focused on loudspeaker drivers. Both shape and topology optimization techniques are applied, with the physics ranging from acoustics, structural mechanics, magnetostatics, and even heat conduction. It is demonstrated how viable engineering solutions, which are not necessarily obtainable via conventional methods, can be found based on the optimization routines for both tweeters, woofers, and compression drivers.

Keywords: Acoustics, shape optimization, topology optimization, loudspeaker, phase plug, waveguide, compression driver

Introduction

Formal optimization techniques, such as shape and topology optimization have reached a level, where they are applicable to engineering problems to a larger degree than ever before. In this work several examples are shown that illustrate shape and topology optimization applied to loudspeaker design cases, such as compression drivers, woofer suspensions, tweeter phase plugs, and magnet systems. The purpose is not to go through all details related to optimization such as filters, algorithms, solver settings, or similar. Nor is it go into an analysis of obtained nominal results for each case; mainly because the designs will never be realized. Also, comparisons between initial/final designs can be misleading: An initial design for a topology optimization may have more volume than an applied volume constraint will allow, and so the initial design may be 'better' in some sense than the optimized design as a consequence of certain constraints. Finally, a comparison between topology optimized designs and shape optimized ditto is also often misplaced, as the latter will often succeed the former, so that the topology is found first, and then the design

is further refined via a shape optimization. In addition, the two methods may not have had comparable constraints. Instead of having a focus on nominal results, the purpose of this paper is to *illustrate, across many different examples, how shape and topology optimization can be applied to multiphysics problems in general, with the loudspeaker serving as a prototypical product.* From here, it is up to the individual engineers to research the topic more, come up with strategies for their application/product, and formulate appropriate objective functions and constraints for the problems at hand.

Theory

Optimization poses the question how best to obtain a certain objective, typically formulated as a minimization problem, given certain constraints, as:

$$\min_{\xi}: \Phi\left(u(\xi), \xi\right) \qquad (1)$$
$$\text{subject to: } c_i(u(\xi), \xi) \leq 0, \qquad i \in \{1, 2, \dots\}$$

where Φ is the objective function, ξ is a vector of design variables, and u is some complex state variable such as for example pressure for an acoustics problem, with constraints c_i.

Shape optimization changes boundary (2D) or surface (3D) shapes to reach a given objective. In COMSOL Multiphysics two different approaches are available: The *Polynomial* approach and the *Free Shape* approach. The former deforms boundaries based on an analytical polynomial, typically of the Bernstein type, and with its close ties to Bezier curves, there can be advantages using this approach for subsequent CAD purposes. The solution space is limited by the polynomials chosen, and while complex shapes can be obtained, the shape can never 'loop back on itself', just as a proper function can only have one output for

one input. As an alternative, the latter *Free Shape* optimization deforms the boundaries based on an underlying PDE, and so in general a larger range of possible solutions exist. However, this approach may not give as smooth shapes as the polynomial approach, and manufacturability can be impacted.

While shape optimization retains the inherent topology of the initial design, *topology optimization* changes a density distribution across a surface (2D) or in a domain (3D) based on an interpolation scheme for the physics in question. This distribution is then interpreted as the optimized topology to be physically realized, see e.g. [1]. It is up to the user to implement the interpolation schemes properly, as optimization is not tied to a specific physics in COMSOL Multiphysics; it is instead its own physics. For static structural mechanics, with accompanying equilibrium and compatibility equations and constitutive relations, the Young's modulus is typically interpolated via the SIMP method [1] as

$$E(\bar{\xi}) = \bar{\xi}^p E \qquad (2)$$

where E is the nominal Young's modulus, $\bar{\xi}$ is a filtered design variable, and p is a penalization factor, typically with value 3 or higher, while for acoustics one appropriate SIMP scheme could be[1]

$$\rho(\bar{\xi}) = \rho_0 + \bar{\xi}^P(\rho_s - \rho_0) \qquad (3)$$

$$K(\bar{\xi}) = K_0 + \bar{\xi}^P(K_s - K_0) \qquad (4)$$

for the density and the bulk modulus, respectively, with the resulting governing acoustics equation being

$$\nabla \cdot \left(\frac{1}{\rho(\bar{\xi})}\nabla \boldsymbol{p}\right) + \frac{\omega^2}{K(\bar{\xi})}\boldsymbol{p} = 0 \qquad (5)$$

with the complex sound pressure $\boldsymbol{p}$. The implied filtering of the design variable ξ is typical both density filtering and projection filtering, which are added to promote binary designs, see [1].

Implementation in COMSOL Multiphysics[2]

Several parameter values, e.g. for the interpolation extremes, scaling parameters, geometry parameters, and more, are input under Parameters.

For shape optimization the node Shape Optimization node is added under Definitions. Either the Polynomial or the Free Shape[3] approach is chosen; for Polynomial, Bernstein was selected. The default Yeoh mesh smoothing was used throughout.

A Topology Optimization node is added under Definitions, with a Projection filter applied using a Projection Slope value initially defined under Parameters, but its value is often ramped up gently in the optimization process. The material interpolations for density and bulk modulus, respectively, are input under Variables as functions of the design variables. These function variables are used as input in a separate Pressure Acoustics node for the design domains only.

An Optimization node is added from the Mathematics selection, and one or more global objectives are added, along with e.g. a Global Inequality Constraint to control the allowed volume fraction.

The SNOPT method was typically used for the shape optimization, while the MMA algorithm was typically chosen for the topology optimization, with the adjoint gradient method activated.

For both shape and topology optimization it is important to set up internal scaling of all optimization parameters, variables, and objectives, and the solver setup in general is not trivial. These details, however, will not be described in this work.

Examples

Several examples have been generated to illustrate the use of shape and topology optimization across different physics, with the physics and optimization type indicated in square brackets under each heading. While an underlying objective function is always present, there is little emphasis on the numerical values obtained, as none of the examples will be realized physically, but instead the focus is on the many application areas for the two types of optimization. Unless otherwise stated, all CAD geometries were created by the author.

Compression Driver and Waveguide
[Acoustic Shape Optimization]

An *internal* acoustics problem was investigated. A three-channel compression driver was to be shape

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[1] A so-called RAMP interpolation [1] was used for the acoustic topology optimization in this work.

[2] This section assumes implementation in v5.5.
[3] The only option for 3D cases

optimized towards an objective of having a smooth sound pressure level response. The example and the procedure are very well described in [2], and reproduced here. Six boundaries have their shape changed in the procedure, see Figure 1, each via a fourth-order Bernstein polynomial, leading to 18 parameters controlling the overall shape of the phase plug. Modifications to the implementation described in [2] have been made to ensure a more robust handling of large deformations of boundaries with less risk of them colliding.
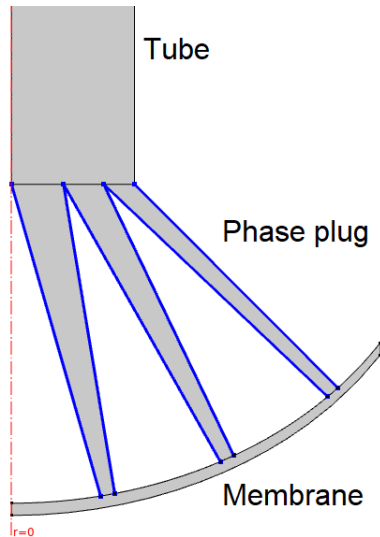
Figure 1: The initial axisymmetric compression driver geometry with the shape optimization boundaries in blue.

The optimized geometry is shown in Figure 2 and the sound pressure levels at the output tube are shown for the initial and the optimized states in Figure 3.
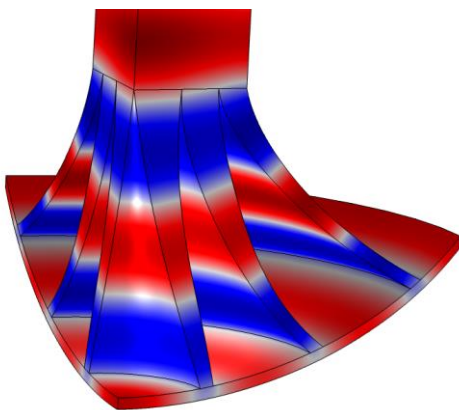

Figure 2: A quarter section of the optimized geometry with the sound pressure field at 16 kHz.

It is seen that a smooth sound pressure level can be obtained with shape optimization even for this

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inherently resonant geometry. Also, in the physical setup there will be thermoviscous damping which will further regulate the pressure response [3].
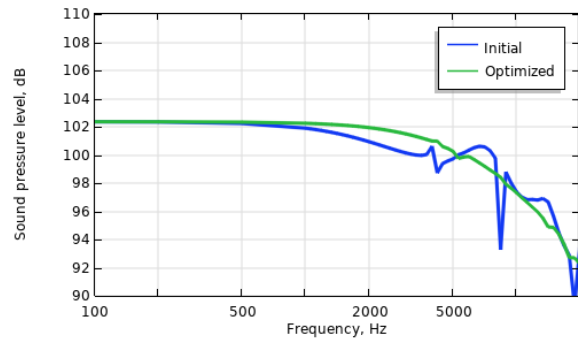

Figure 3: The sound pressure levels in the tube for the initial state (blue) and for the optimized (green) state.

A similar approach was applied to an *external* 3D problem using the *Free Shape* type of shape optimization; a tweeter waveguide with a horizontal DI[4] and a vertical ditto in the objective function. Due to space constrictions all but the optimized geometry, Figure 4, is left out of this work.
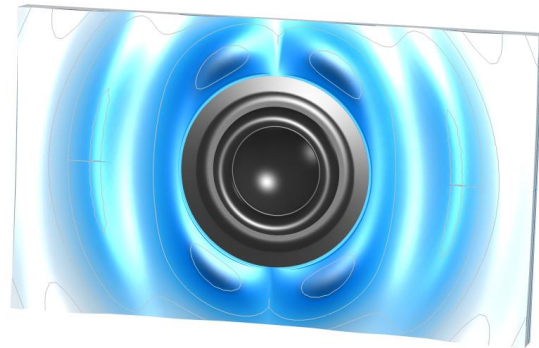

Figure 4: Shape optimized tweeter waveguide with the tweeter in black and coloring and contouring to indicate the shape.

Woofer Cone
[Vibro-acoustic Shape Optimization]

A coupled structural mechanics – acoustics problem was investigated. A woofer model was made, which is complete in that a voltage is applied as an input and the electromagnetics coupling is included via a lumped parameter circuit. It was decided to apply shape optimization to the cone part of the woofer. The air can be included explicitly with mesh and physics, but the shape optimization solver setup becomes quite involved when deformed boundaries

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[4] Directional index.

are at the interface between two physics, such as is the case here with the cone boundaries interfacing the acoustic and the structural domains. Instead, the author opted for including the air via an added air loading mass and a Rayleigh integral [4] for the calculation of the sound pressure level:

$$\boldsymbol{p}(P) = \frac{-\omega^2 \rho_0}{2\pi} \int_S \boldsymbol{w}(Q) \frac{e^{-ikR}}{R} dS, \qquad (6)$$

where $\boldsymbol{p}(P)$ is the complex pressure in the observation point P and $\boldsymbol{w}(Q)$ is the complex axial surface displacement in the point Q. The Rayleigh integral will only evaluate the sound pressure level accurately when the surface in question is perfectly flat and placed in a baffle, and while the baffle indeed is included the simulation, the loudspeaker surface is not flat. However, the errors made are not too significant as to interfere with the overall trend of the sound pressure level response. Also, the difference between the sound pressure level found using e.g. an External Field calculation and by applying the Rayleigh integral can be calculated once for the initial geometry, and if the optimization procedure does not significantly change the surface layout, then the difference can be incorporated into the procedure in advance, so that the target level corresponds to the actual External Field response.

With this implementation the deforming boundaries exist solely on the solid mechanics physics, and the solver setup can be kept the same as for all shape optimization cases considered in this work. Third-order Bernstein polynomials were used for the deforming boundaries. The geometry is shown in Figure 5 with the initial and the optimized cone for an objective towards flattening the sound pressure level.
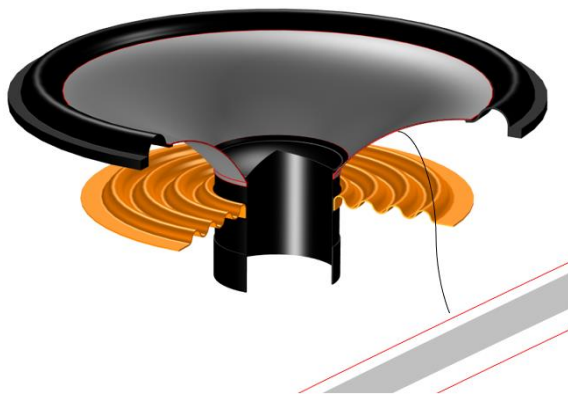

Figure 5: The woofer geometry with a zoom of the initial cone (red outline) and the optimized cone (gray).

The sound pressure level response in 1 m distance was calculated via the Rayleigh integral during the optimization, and the initial and the optimized responses are shown in Figure 6. The optimized sound pressure level response has been lifted somewhat at higher frequencies. There is of course more to a loudspeaker than on-axis response, and so for example the off-axis behavior of the optimized geometry should be investigated. Also, the optimized cone does not have a constant thickness, which may pose a manufacturing problem. However, this case demonstrates how solid mechanics parts can be designed using shape optimization. Also, it is recommended to apply Phase Decomposition [4] on the initial and the optimized geometries to further explore the effects of the optimization.
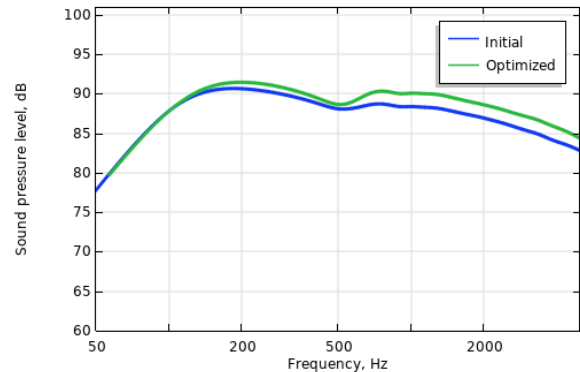

Figure 6: The initial (blue) and the optimized (green) sound pressure level response.

Woofer Magnet System
[Magnetic Shape Optimization]

With the starting point being the Application Library model 'Topology Optimization of a Magnetic Circuit' a *shape* optimization approach was instead taken, as the original optimization in fact did not change the initial topology. A loudspeaker magnet system with a permanent magnet and soft iron core domains is described via Maxwell's equations and constitutive relations. It is to be shape optimized taking into consideration the non-linear BH-curve of the soft iron domains.

An 8th order Bernstein polynomial was applied to a single boundary indicated in Figure 7. The objective is to maximize the Bl-factor, and no explicit constraints are added other than a maximum displacement limit for the boundary.
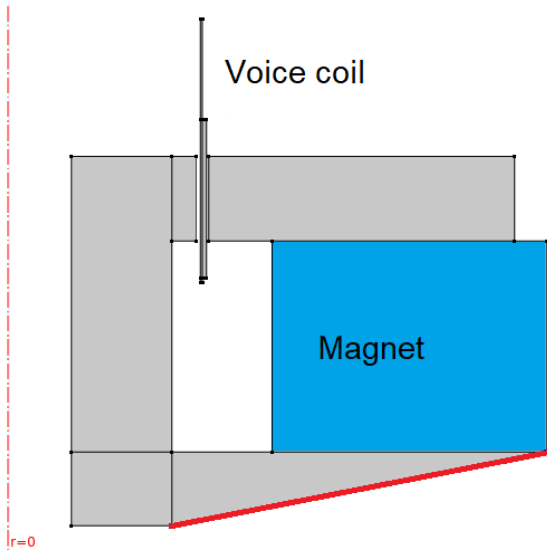
Figure 7: The axisymmetric magnet system with soft iron domains in gray, and the shape optimization boundary in red.

The optimized design is shown in Figure 8, where the resulting shape resembles that of the original application model, but with a higher Bl-factor than the topology optimized cases in said model, even with their constraints removed. This is typical, as many topology optimized designs can benefit from an additional shape optimization once the topology is established.
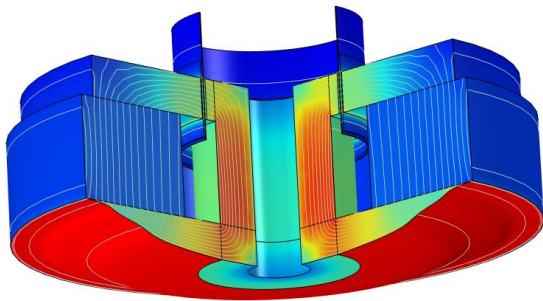

Figure 8: The shape optimized design of the lower soft iron core, with magnetic flux density color map and magnetic field contour lines. The red surface is shape optimized.

Woofer basket I
[Structural Mechanics Topology Optimization]

As a structural mechanics example, the basket of a woofer was investigated. In general, is it desired that the basket is as stiff as possible, so that the reaction forces from the loudspeaker driver do not lead to

elastic deformation of the basket, but instead go into accelerating the much heavier cabinet. A stationary analysis with topology optimization included was run with an axial stiffness objective[5] defined. A constraint was put on the optimization that only 50 % of the initial material remains after the optimization. The basket was fixed at the top, and a distributed axial force was applied to the bottom surface of the basket. The optimized result is shown in Figure 9. The darker domains were not allowed to change in the optimization procedure, while the lighter domains (and the 12 holes) are a result of the topology optimization, as the initial design was completely closed. The design was based on a quarter of the full 3D basket, so symmetry conditions were applied, and are inherently present in the resulting 3D design.
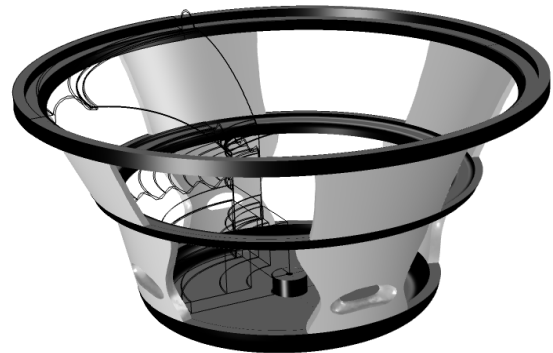

Figure 9: The topology optimized basket with lighter domain having been optimized while darker domains were fixed. 8 large and 4 small holes have appeared after the optimization.

For a real-world basket in a loudspeaker company, the analysis should be done across the frequency range of operation, but here only a static analysis was run to illustrate the optimization procedure. Also, different load cases can be added to, for example, take gravity into account for larger woofers.

Woofer basket II
[Structural Mechanics Shape Optimization]

The topology optimized design served as inspiration for a simpler initial design with 8 holes and straight sides for a shape optimization routine with the same objective. The result is shown in Figure 10. The initial design is shown in black, while the gray overlay shows the optimized geometry. A maximum allowed displacement was set, somewhat arbitrarily,

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[5] As an alternative to the more common strain energy minimization, based on prior knowledge.

to around 60 % of the initial thickness of the ribs. Just as with the topology optimization, certain boundaries are excluded from the optimization, since the spider and surround need to land on flat surfaces. However, the design has changed from the initial state in that the ribs are now thicker and rounder. Again, one should consider manufacturability, as the effort of changing the basket may not be worth it if it is more difficult produce. However, both the topology optimization and the shape optimization results can give the designers new ideas that can be realized in a simpler way.
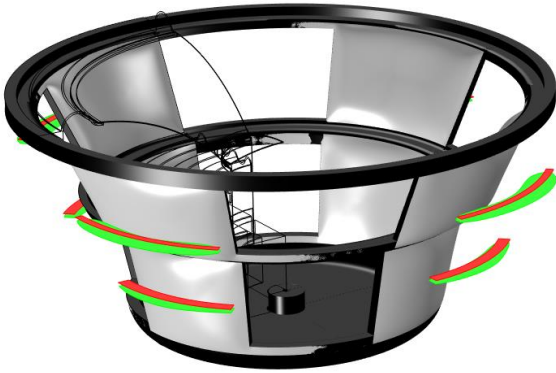

Figure 10: The shape optimized basket with lighter domain having been optimized and overlaying the initial design in black. The colored (and transversely offset) surfaces illustrate the thickness of the basket before (red) and after shape optimization (green) at two heights.

As an alternative, *shell* shape optimization[6] is available in COMSOL Multiphysics and should also be considered for basket cases[7]. This method can lead to corrugated designs, and so manufacturability needs to be considered.

Woofer Heat Sink
 [Heat Conduction (Solid) Topology Optimization]

A large part of the electrical energy applied to a loudspeaker is converted to thermal energy, and the resulting heat can lead to unwanted thermal compression. The excessive heat can also change the elastic properties of the moving parts, thus changing the nominal Thiele-Small parameters. With Fourier's Law as the governing equation giving the temperature T

$$-\nabla \cdot (k\nabla \mathrm{T}) = Q \qquad (7)$$

with k being the thermal conductivity and Q being a volumetric heat source, a steady state heat conduction problem was set up to design a basket with a heat sink appendix via topology optimization. With the voice coil assumed the main source of heat, a topology optimization setup was made with an objective to minimize the thermal resistivity [5] over the optimization domain Ω described as

$$\Phi = \int_{\Omega} k(\bar{\xi})(\nabla \mathrm{T})^2 d\Omega \qquad (8)$$

with an interpolated thermal conductivity $k(\bar{\xi})$. More details can be found in [5] and [6]. The topology optimized heat sink design is seen in Figure 11. The optimization was carried out for a quarter of the full 3D geometry. The resulting geometry is a consequence of mesh size, density and projection filtering, defined optimization domain, and other specifics of the setup, and again the actual geometry is of less importance than illustrating the functionality itself.
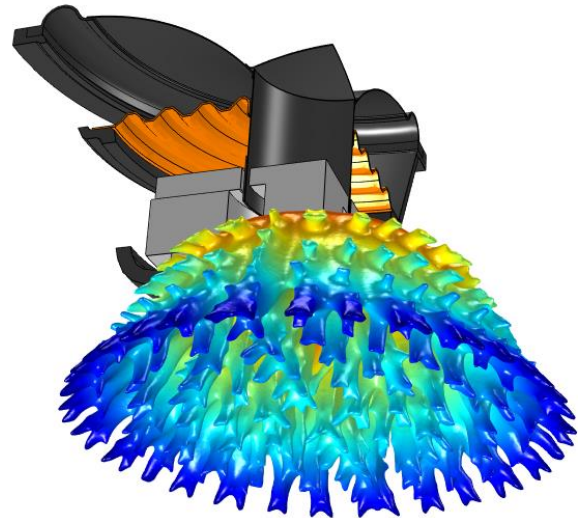

Figure 11: Topology optimized heat sink with temperature distribution.

Tweeter Phase Plug
                    [Acoustic Topology Optimization]

This example has already been shown in [1], but is included for completeness, combining acoustics and topology optimization. A tweeter was designed being fully coupled with the electromagnetic properties included as lumped parameters. The initial on-axis frequency response was found to have a valley across the frequency range of interest and a flatter response was sought via topology optimization.

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[6] See for example Application Library example "Shape Optimization of a Shell".

[7] No pun intended.

The topology optimized geometry is shown in Figure 12 with the resulting phase plug shown in grey. Due to the cylindrical symmetry of the tweeter the on-axis was found out via a 2D axisymmetric setup.
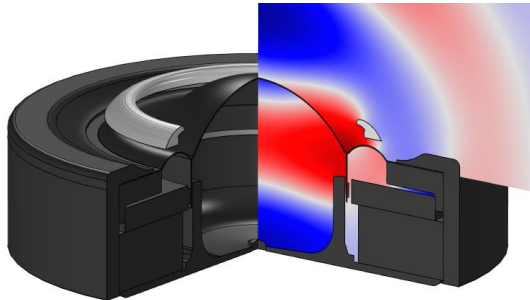

Figure 12: The topology optimized phase plug is shown in grey while the tweeter is black, with the sound pressure at 16 kHz indicated.

The initial and the optimized on-axis sound pressure levels are shown in Figure 13, where it is seen that the sound pressure level has been flattened across the frequency range of interest after the optimization.
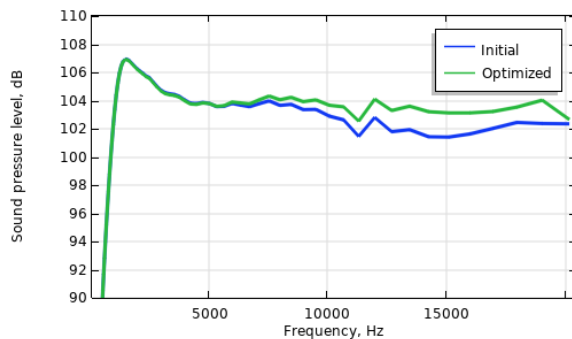

Figure 13: On-axis pressure in linear frequency scale, before (blue) and after optimization (green). The frequency response has been flattened by the topology optimization.

Discussion

Optimization in general should never replace proper engineering in any way. However, for certain problems, trial-and-error methods or combinatorics approaches are too time-consuming to produce viable results. Here, shape and topology optimization can be invaluable in making the correct engineering decisions. Also, while manufacturability may be an issue for some optimized results, there can still be learnings in the result, which can lead to certain insights about product behavior, which in turn can result in simpler design changes. As with any tool, optimization should not be 'the hammer to all nails', but instead a powerful addition to the toolbox.

Conclusion

It has been demonstrated across a wide range of physics how formal shape and topology optimization methods can be utilized in the design of loudspeaker drivers. Some of the obtained optimized designs are non-intuitive, and probably would not have been found using traditional engineering methods. While the focus here has been on the loudspeaker, the combinations of physics and optimization methods will be applicable across a wide range of products. Therefore, engineers are encouraged to investigate these methods more to see how they best can be applied in the industry.

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