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# Intraplate Stress Analysis by COMSOL Multiphysics

G. Swetha, G. Pavankumar* and A. Manglik
CSIR-National Geophysical Research Institute
* Corresponding author: National Geophysical Research Institute, Uppal Road, Hyderabad – 500660, INDIA (gayatripavan@ngri.res.in)

**Abstract:** Mathematical modeling tools are extensively used in geosciences to delineate the earth structure at various spatial scales as well as to simulate coupled earth processes involving multiphysics concepts. In the present work, we demonstrate an application of COMSOL for the computation of elastic intraplate stresses in the continental crust. Understanding the stress state of continental crust is important for earthquake studies. We have computed intraplate stresses due to density heterogeneities and mechanical property variations for a model of the continental crust, delineated by geophysical techniques along a profile in the western part of the Indian shield, and validated our computations with the published results. The finite element model consists of 11 layers, each having its own geophysical significance and distinct material properties. The computed results show the significant stress concentration in the crust. The stress computation protocol prepared for this work may be used to carry out similar exercises under the Earth Science module.

**Keywords:** Intraplate stress, Continental crust, Plain stress, multiphysics

## 1. Introduction

The earth system is a highly complex and nonlinear system comprising physical processes at multi-scales. These processes are studied using various physical laws involving physical parameters that need to be estimated mainly from geophysical observations at or above the surface of the earth. The complexity of the earth system warrants the use of advance numerical techniques to construct mathematical models as close to the reality as feasible for improved understanding of earth processes. Advanced numerical simulation tools are being extensively used in geosciences to delineate the earth structure at various spatial scales and estimate physical properties as well as to simulate coupled earth processes involving multiphysics concepts.

In the present work, we demonstrate an application of COMSOL for the computation of two dimensional (2-D) elastic intraplate stresses in the continental crust, the uppermost about 35 km thick layer of the earth in the continental region. The crustal rocks in the intraplate continental regions show elastic behavior in the upper 10-20 km and ductile behavior below this depth range. Estimation of intraplate stresses and identification of zones of stress concentration is significant in seismology for the understanding of processes leading to earthquakes. Although plate boundary regions such as the Himalayan collision zone and the Andaman-Sumatra subduction zone are seismically more active than the continental interiors and COMSOL can be used to model stresses in these tectonic settings, we have chosen an example from the NW part of the Indian shield basically to validate the present results with the earlier published one (Manglik et al., 2009). We have computed stresses in the continental crust, induced by density heterogeneities and mechanical property variations, along the NW-SE trending Nagaur-Jhalawar profile. The model of the continental crust was delineated by integrated geophysical techniques (controlled source seismic and gravity methods). We have analyzed elastic intraplate stresses which might generate in the crust due to the complex crustal structure for different scenarios of elastic properties of various crustal layers.

## 2. Crustal Structure

A 400km long NW-SE trending deep crustal seismic profile, the Nagaur-Jhalawar transect, was shot across the Aravalli-Delhi Fold Belt (ADFB) to delineate the crustal structure and infer about the tectonics of this region (Figure 1). This belt occupies the northwestern part of the Indian shield and contains a record of varied geological processes and tectonic events since 3500 Ma. ADFB represents one of the oldest nuclei related to the evolution of the Indian continental crust. It is considered as a chain of Relic Mountains extending in the NE-SW direction for about 700 km and consisting of intensely folded, deformed and metamorphosed Proterozoic rocks overlying the Archaean gneissic basement. The fold belt is bounded in the NW by neo-Proterozoic Marwar Basin (MB) and in the SE by meso-neo-Proterozoic Vindhyan Basin. Aravalli sequences and the Vindhyan basin are separated by the Great Boundary Fault. This NE-SW trending fault zone with a width of 10-20 km extends up to the Moho (base of the continental crust). The Marwar Basin, the youngest Proterozoic sequence, consists of flat, un-deformed clay evaporate sequences, sandstones and slit stones overlying the Erinpura granites and the Malani igneous suite.

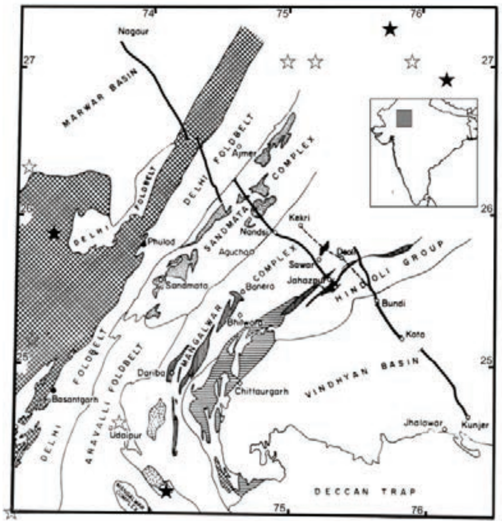

**Figure 1.** (a) Location of the Nagaur-Jhalawar transect across the Aravalli-Delhi Fold Belt in NW India. Open and filled stars are the epicenters of historical and recent earthquakes, respectively (after Manglik et al., 2009).

The transect starts from Nagaur in the MB, cuts across the Vindhyan Basin. Further geophysical investigations incorporating magnetotelluric and gravity studies were carried out along this transect to develop a model of the tectonic evolution of the region. These studies revealed the complex nature of the sub-crustal structure, including the presence of a high density intrusive forming a dome-shaped structure in the mid-to-lower crust beneath the Delhi Fold Belt (Figure 2). The complex nature of the crustal structure with significant lateral density variations present a suitable scenario for the generation of large stresses in the crust.

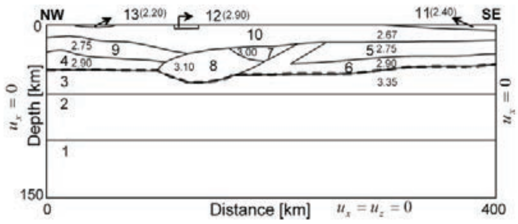

**Figure 2.** A simplified crustal structure along the profile, derived from geophysical (seismic and gravity) data, along with the boundary conditions (after Manglik et al., 2009). Dashed curve marks the base of the crust. Numbers in small fonts represent density values in kg/m$^3$. This model is used to compute intraplate stresses by COMSOL.

We have analyzed elastic intraplate stresses which might generate in the crust due to such a complex structure for different scenarios of elastic properties of various crustal layers. Although this region has not shown a significant level of intraplate seismicity, it is worthwhile to investigate the stress state of the region to understand the role of the dome-shaped structure in inducing local stresses.

## 3. Stress Analysis by COMSOL

The crustal structure (Figure 2) is sub-divided into 13 patches based on the density of various blocks representing different rock types. The first two patches (1 and 2) are fictitious layers included in the model to minimize the artifact of the bottom boundary conditions on the computed stresses within the crust. We have also ignored the patch numbers 12 and 13 as these are too small to significantly influence the stresses generated by the deeper structures such as the domal structure (patch number 8). Thus, the finite element model consists of 11 patches. The model geometry, shown in figure 2, was constructed using CAD software and then imported into Model Builder module of COMSOL. The geometry of the model can as well be constructed within the graphical user interface of COMSOL.

Next, we have defined and set the material properties for every patch in the material section. In geophysical applications, we normally don't have the measured values of the elastic properties. Instead, these are estimated from the inferred seismic velocity and density values. We have used the following relationship to estimate the Young's modulus $E$ (Pauselli and Federico, 2003; Manglik et al., 2008)

$$E = \rho V_P^2 \frac{(1+\nu)(1-2\nu)}{3\nu}, \quad (1)$$

where $V_P$, $\rho$, and $\nu$ are the seismic P-wave velocity, density and Poisson ratio, respectively. The estimated values of $E$ for our geophysical model fall in the range of $0.1 \times 10^{10}$ to $1.8 \times 10^{11}$Pa. As the information about the shear wave velocity is not available along the profile, it is difficult to estimate the values of the Poisson ratio. Therefore, we assume its values for different patches based on general geophysical understanding of crustal rocks and thermal structure in continental crusts. The material properties for various patches are listed in table 1. For the fictitious patched (1 and 2), we have assumed both $E$ and $\nu$ in such a way that these represent extremely weak layers (very large Poisson ratio).

The governing physics of the problem and boundary conditions are then added to the geometry under the application mode. A single geometry may contain multiple application modes that are intrinsically linked. However, in the present analysis we confine only to elastic stress modeling which is governed by the following equation of equilibrium and Hookes's law for an isotropic medium

$$\nabla \cdot \bar{\bar{\sigma}} = -\rho \bar{g}, \quad (2)$$

where

$$\bar{\bar{\sigma}} = \lambda(\nabla \cdot \bar{u})\bar{\bar{I}} + \mu(\nabla\bar{u} + \nabla\bar{u}^T), \quad (3)$$

Here, $\bar{\bar{\sigma}}$ is the stress tensor, $\bar{u}$ is elastic displacement, $\bar{g}$ is gravity acceleration, and $\rho$ is density. $\lambda, \mu$ are elasticity parameters called Lame's parameters. The elastic displacement $\bar{u}(u_x, u_y, u_z)$ satisfies the bi-harmonic equation

$$\nabla^4 \bar{u} = 0. \quad (4)$$

The boundary conditions used for the analysis are shown in figure 2. The top boundary is considered as the free surface whereas the bottom boundary is the fixed boundary $(u_x = u_z = 0)$. We assume no horizontal displacement $(u_x = 0)$ across the left and right boundaries.

An additional step for geophysical problems is to include the effect of gravity induced body load in the model. We have added this body load parameter under the solid mechanics for each patch. The density values prescribed under the material properties for every patch were used to compute this body load. There is no other externally applied body load in our model. We have used plane stress formulation.

The last step towards building the finite element model for analysis was to mesh the geometry for which we have used triangular elements under the mesh section with automatic meshing option.

**Table 1.** Elastic properties assigned to various patches of the model

| Patch Nr. | Model 1 | | Model 2 | |
|---|---|---|---|---|
| | $E$ $[10^{10}\text{Pa}]$ | $\nu$ | $E$ $[10^{10}\text{Pa}]$ | $\nu$ |
| 1 | 0.1 | 0.499 | 0.1 | 0.499 |
| 2 | 0.8 | 0.46 | 0.8 | 0.46 |
| 3 | 6.0 | 0.35 | 18.0 | 0.25 |
| 4 | 1.0 | 0.25 | 8.0 | 0.25 |
| 5 | 2.0 | 0.25 | 2.0 | 0.25 |
| 6 | 1.0 | 0.25 | 8.0 | 0.25 |
| 7 | 12.0 | 0.25 | 1.0 | 0.35 |
| 8 | 12.0 | 0.25 | 1.0 | 0.35 |
| 9 | 2.0 | 0.25 | 2.0 | 0.25 |
| 10 | 6.5 | 0.25 | 6.5 | 0.25 |
| 11 | 2.0 | 0.25 | 2.0 | 0.25 |

## 4. Results

Manglik et al. (2009) computed various model cases and analyzed the level of stress concentration due to the lateral variations in the crustal structure of ADFB to explore the possibility of a suitable model case that yields the least stress concentration to corroborate with the low level of seismicity in this region. Here, we have considered their two model cases (Table 1) for the intraplate stress analysis by COMSOL. In the first model, the dome-shaped structure is mechanically strong (patches 7 and 8) and is embedded in a mechanically weak lower crust (patches 6 and 8). The sub-crustal layer (patch 3) is also mechanically relatively weak compared to the domal structure. For this model, we have obtained large stress concentration within the domal structure as well as at its contact with the lower crust, especially at the profile distance of about 160 km (Figure 3a). The shear stress in the near surface region above the domal structure also shows significant variations.

The second model incorporates mechanically strong lower crust and weak domal structure. The sub-crustal layer in this model is stronger than that used in model 1. The results (Figure 3a) show an enhancement in the shear stress at the junction between the lower crust and the left edge of the domal structure at the profile distance of about 100 km whereas the stress concentration seen at the profile distance of 160 km in the previous model now vanishes. Instead, we observe stress concentration at the left edge of the patch 5 at the distance of 220 km. The magnitude of shear stresses within the domal structure is less for this model compared to the previous model.

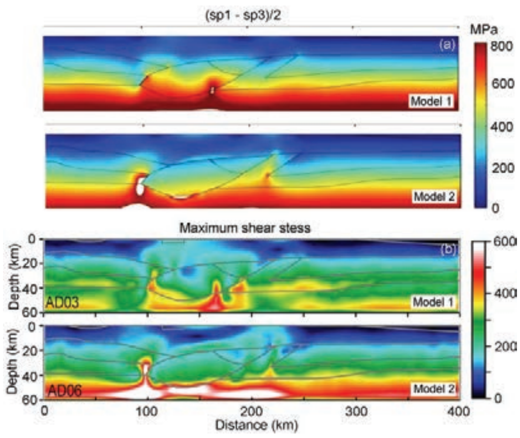


**Figure 3.** A comparison of maximum shear stress along the profile computed by using COMSOL (a) with the results published by Manglik et al. (2009) (b).

We compare the present results, obtained by COMSOL, with those published by Manglik et al. (2009) to validate our results. We observe that although the general pattern of the shear stresses by present analysis is similar to the results of Manglik et al. (2009) (Figure 3b), our results show systematically larger magnitude for stresses and small-scale variations seen in their results are not visible in present results. One possibility could be inadequate grid resolution. We have computed our results after grid refinement and found consistent results. Therefore, the grid resolution problem is ruled out. Future work is needed to resolve this discrepancy in results.

## 5. Conclusions

The computed results show that the domal structure and the lower crust control the stress concentration in the upper crust. The results show that the crustal structure along the Nagaur–Jhalawar profile, as delineated by seismic and gravity studies, should induce significant intraplate stresses in the crust, especially in the complex tectonic zone demarcated by domal high velocity structure. The mechanical properties of these structures influence the stress state in the shallow part of the crust and lead to stress concentration in various parts of the profile. These results, in corroboration with measured stress data when available, can help in constraining models of the crust of this region.

The analysis carried out here is an example for the usage of COMSOL to model elastic intraplate stresses due to density and mechanical property heterogeneities within continental crust. The stress computation protocol prepared for this work may be used to carry out similar exercises under the Earth Science module. However, the discrepancy between the present and the published results need to be resolved.

## 6. References


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