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# A Study of Fluid Flow and Heat Transfer in a Liquid Metal in a Backward-Facing Step under Combined Electric and Magnetic Fields

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**Abstract: This study used COMSOL Multiphysics as the analytical tool to investigate the effect of applied magnetic and electric fields on the flow phenomena of an electrically conducting fluid in a backward step configuration. The magnetohydrodynamic (MHD) approach was validated by comparison with the existing solution for the Hartmann problem while the back step flow was validated by comparison with previously obtained solutions. The implementation of the magnetic and electric field has significant impact on the effective Reynolds number and thus leads to changes in the separation and reattachment point downstream the step. Depending on the strength of the fields, the recirculation regions downstream could become smaller/vanish due to the change in the pressure and velocity distribution. In addition, this change in the velocity profile due to the MHD effect in a back-step flow also affects the heat transfer mechanism in the flow since convection is very dependent on the velocity distribution.**

**Keywords: MHD, Magnetohydrodynamic, Back-step flow, heat transfer, separation**

## 1. Introduction

### 1.1 Background

Liquid metal flows subjected to combine electric and magnetic fields are part of a larger study, which is known as the Magneto-hydrodynamics (MHD). The concept of MHD is that the magnetic fields can induce currents in a conductive fluid that creates force, which will affect the flow and may even change the magnetic field itself. The study of MHD has become very important because of its growing applications. For instance, MHD pumps are utilized for different purposes, including liquid metal cooling. One of the primary products employing this process is the liquid metal cooled nuclear reactor, which is used in nuclear submarines as well as many power generation applications. Some other potential liquid metal MHD applications include, but are not limited to, energy conversion technology and metallurgy. A liquid-metal MHD power converter has been successfully operated with the generation of AC electrical power [4].

Due to its wide potential, a basic understanding of the MHD phenomenon is essential. The so-called Hartmann flow has been studied extensively. The Hartmann flow is the steady flow of an electrically conducting fluid between two parallel walls under the effect of a normal magnetic and electric fields. However, many flows are not between parallel walls. Multiple engineering applications such as flow in diffusers, airfoils with separations, combustor, turbine blades and many other relevant systems exhibit the behavior of separated/reattached flows. Since the effect the magnetic and electric fields on flow patterns may be significant, it is worthwhile to specifically study their effect on separated flows. The goal of this project is to investigate the effect of applied magnetic and electric fields on the flow over the backward facing step.

### 1.2 Background on flow pattern over a backward facing step

Flow over a backward facing step is an example of unilateral sudden expansion, which results in flow separations and reattachments. Figure 1[1] shows the schematic of a backward facing step flow:

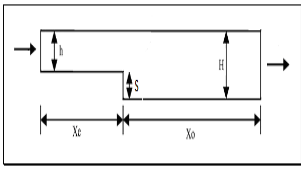

**Figure 1[1]: Schematic of backward facing step geometry (not to scale)**

This project will focus on the laminar regime of backward facing step flow. Without the effects of magnetic and electric fields, the behavior of the flow over a back facing step in laminar regime is very dependent on the Reynolds number and the ratio between the step height (S) to the duct height (H). For laminar flow, various re-circulation zones occur downstream from the step, as shown schematically in Figure 2[2]. As the Reynolds number of the flow increases, the first region of separation occurs at the step to x2 on the bottom wall (Zone A). Next, the second region of separation occurs between x4 and x5 on the top wall (Zone B). As the Reynolds number increases into the transition zone, a third separation region occurs in (Zone C) on the bottom wall. Theoretically, recirculation zones will continue to develop downstream as the Reynolds number increases and the flow remains laminar. However, this has not been observed experimentally and the flow will eventually become turbulent.

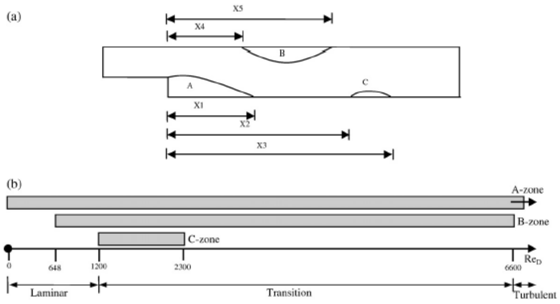

**Figure 2[2]: Three recirculation zones for laminar flow**

## 2. Methodology

COMSOL Multi-physics was used in this study to investigate the effect of magnetic and electric field on the flow pattern over a back facing step flow with liquid metal. This was approached as a two-step process. First, the implementation of magnetic and electric fields in COMSOL needs to be validated using the Hartmann problem. Secondly, the validation of a typical backward-facing flow (without the effect of magnetic and electric field) is performed in COMSOL. The successful implementation of the two models would allow the investigation of the MHD effect on the backward step flow.

### 2.1 Hartmann Problem Theory

The Hartmann problem is one of the simplest problems in Magnetohydrodynamics. However, it gives insight into MHD generators, pumps, flow meters and bearings. It concerns the steady viscous laminar flow of an electrically conducting liquid between two parallel plates under the effect of imposed magnetic and electric fields (Figure 3).

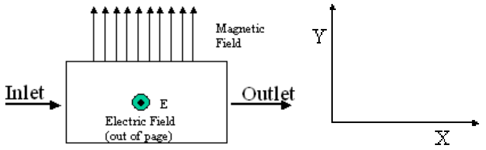

**Figure 3: Hartmann flow in a flat channel with imposed electric and magnetic field**

The constant magnetic field acting in the +Y direction and the electric field acting in the Z direction are the external set parameters know as $B_0$ and $E_Z$.

The flow of an incompressible fluid between parallel plates is governed by the equation of continuity [3]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

and the Navier-Stokes (momentum) equations which have the following form [3]

$$0 = \frac{-\partial P}{\partial x} + \mu_f \frac{\partial^2 u}{\partial y^2} - F_X$$

$$0 = \frac{-\partial P}{\partial y} + \mu_f \frac{\partial^2 v}{\partial y^2} - F_Y$$

$$0 = \frac{-\partial P}{\partial z} + \mu_f \frac{\partial^2 w}{\partial y^2} - F_Z$$

Where $\frac{-\partial P}{\partial x}$, $\frac{-\partial P}{\partial y}$, $\frac{-\partial P}{\partial z}$ are the components of the pressure gradient in the X, Y and Z directions respectively, $\mu_f$ is the dynamic viscosity of the fluid, $F_X$, $F_Y$ and $F_Z$ are the force components in the X, Y and Z directions, which are zero in the simple Poiseuille flow in the absence of gravity (which is the case here). However, in the case of applied magnetic and electric field, the force in the X, Y, and Z direction is known as the Lorentz force. The Lorentz force is due to induced/imposed current and the imposed magnetic field. With the magnetic and electric field, the Navier-Stokes equations become [4].

$$0 = \frac{-\partial P}{\partial x} + \mu_f \frac{\partial^2 u}{\partial y^2} - J_Z B_0$$

$$0 = \frac{-\partial P}{\partial y} + J_Z B_X - J_X B_Z$$

$$0 = \frac{-\partial P}{\partial z} + \mu_f \frac{\partial^2 w}{\partial y^2} - J_X B_0$$

Where $J_X$ and $J_Z$ are the current density components [4]:

$$J_X = \sigma(E_X - B_0 w)$$

$$J_Z = \sigma(E_Z - B_0 u)$$

$\sigma$ stands for the electrical conductivity of the liquid metal. Here $E_X$ and $E_Z$ represent the electric field components in the X and Z direction.

From the schematic diagram of the problem under study, the only relevant Navier-Stokes equation that remains is the following since the applied external electric field is only in the Z direction and the velocity in the Z direction is 0 [4,5]:

$$0 = \frac{-\partial P}{\partial x} + \mu_f \frac{\partial^2 u}{\partial y^2} - \sigma(E_Z + B_0 u) B_0$$

With this equations and the assumption of no slip condition at the wall, the analytical solution is [4,5]:

$$u = \frac{{y_0}^2}{M^2}\left( \left(\frac{1}{\mu_f}\frac{\partial P}{\partial x} + \frac{M}{y_0}\sqrt{\frac{\sigma}{\mu_f}} E_Z\right) \right)\left( \frac{(ch(My/y_0))}{chM} - 1 \right)$$

Where the Hartmann number is given by $M = y_0 B_0 \sqrt{\sigma / \mu_f}$. Here ch and sh denote the hyperbolic cosine and sine, respectively.

### 2.2 Validating flow over a backward step in the absence of external force

There is no known exact solution for a flow over a backward step. However, much experimental data have been published [1]. The experimental data show the separation and the reattachment point of the sudden expansion based on the Reynolds number and the size of the step. The basic governing equations for flow over a step are the stationary incompressible Navier-Stokes equations [6,7]:

$$-\mu_f \nabla^2 u + \rho(u \cdot \nabla) u + \nabla p = F$$

And the equation of continuity [5]:

$$\nabla \cdot u = 0$$

The first equation is the momentum balance equation from Newton's second law. The second equation is the equation of continuity, which implies that the fluid is incompressible. Since flow over a backward step has been a common benchmark problem in CFD, the COMSOL library already has a model for it. However, the properties the model used are different than the liquid metal properties. The actual properties used by the COMSOL library are air properties. The results for the COMSOL model using air properties have been validated against the experimental data [1] given the same geometry and Reynolds number. The reattachment and the separation points are consistent with these obtained from experiments. In order to validate the model for NaK, the liquid metal properties will be used in the COMSOL model. The model can be validated if the separation and reattachment points are the same as the ones produced with the air properties for the same Re.

## 3. Validation

### 3.1 Validation of Hartmann Flow in COMSOL Multiphysics

The implementation of Hartmann flow in COMSOL is the first step. The magnetic field is implemented in the Magnetostatics module. To obtain the magnetic flux in the +Y direction, constant magnetic field is applied in all 4 boundaries as shown in Figure 4 and the electric conductivity of the liquid metal is input into the sub-domain physics.

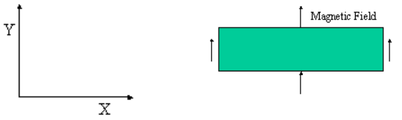

**Figure 4: Boundary condition for the Magetostatics Module**

The solution obtained by using the Magnetostatic module in COMSOL is shown in Fig 5.

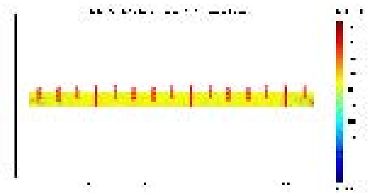

**Figure 5: Magnetic field applied in the Y direction**

The resultant magnetic flux density is then used to compute the Lorentz force acting against the flow. The Lorentz force depends on the velocity, external electric and magnetic field. The Hartmann number, $M = y_0 B_0 \sqrt{\sigma / \mu_f}$ , only depends on the strength of the magnetic field given the fluid properties. Therefore, it is sufficient to apply the magnetic field in order to validate the implementation of the Hartmann problem. In addition, the velocity profile at the inlet was modeled as a parabolic shape laminar flow with no slip condition applied at the wall. The pressure gradient is the main driver of the flow. Table 1 contains the geometry dimension for the model. The values for the height and the length of the channel are chosen to be the same as the ones input into the analytical calculations. The mesh size is chosen based on its accuracy and computing time. A coarse mesh would not be able to generate good results while a finer mesh would increase the computing time of the model.

**Table 1: Geometry of the Channel and Mesh**

| | | |
|---|---|---|
| Height of channel | H | 0.2 m |
| Length of channel | L | 2 m |
| Mesh Size | Number of degrees of freedom | 5207 |

With these settings, COMSOL is able to reproduce the analytical solutions given a constant pressure gradient, fluid properties and Hartmann number. Figure 6 shows the overlay between the analytical and COMSOL solution. The difference between the absolute values is within 1%.

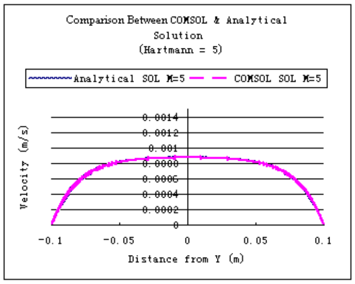

**Figure 6: The COMSOL solution compares well with the analytical solution**

With the increase of the Hartmann number, the absolute maximum velocity at the center of the channel decreases while the velocity near the walls increases. This is so because the Lorentz force acts in the negative X-direction an so as to oppose the flow (see Figure 7).

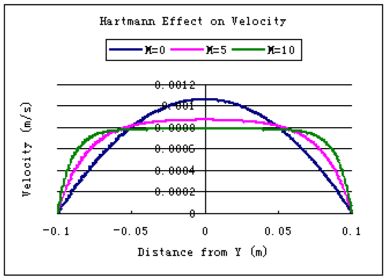

**Figure 7: Effect of increasing Hartmann number on the velocity profile**

The velocity gradient near the wall is much bigger due to the combined effect of the Lorentz force and the no slip condition. The magnitude of the Lorentz force is a function of the incoming velocity. Greater velocity will result in a greater Lorentz force. Figure 8 shows the effect of Hartmann number (M=0, M=5 and M=10) on the normalized velocity profile of the flow for both the analytical solution and the solution from COMSOL; the agreement is excellent.

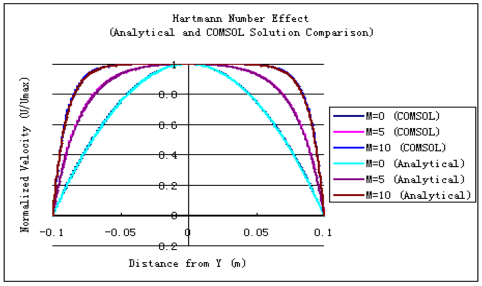

**Figure 8: Hartmann number effect (analytical and COMSOL solutions comparison)**

### 3.2 Validation of Backward Step Flow in COMSOL using liquid metal properties

The backward step flow problem has already been modeled in COMSOL and is included in the Model Library. Fluid enters from the left side of the channel with a parabolic velocity profile, passes over a step and then leaves through the right side of the channel as shown in Figure 9. No slip conditions are assumed at the upper and bottom of the channel and a fully developed parabolic laminar flow velocity profile is imposed at the inlet.

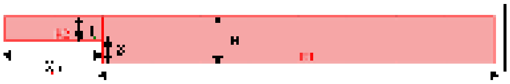

**Figure 9: The back-step geometry**

This geometry has an expansion ration, ER, of 1.942, which is consistent with the literature[2]. In the model, the following geometry dimensions are assumed.

**Table 2: Backward facing dimension in the model (meters)**

| | | |
|---|---|---|
| Height of inlet channel | h | 0.0052 |
| Height of outlet | H | 0.0101 |
| Step height | S | 0.0049 |
| Inlet channel length | Xe | 0.02 |
| Outlet channel length | Xo | 0.08 |
| Number of degrees of freedom | (Fig 14) | 19722 |

The Reynolds number is defined as, $\mathrm{Re} = \frac{uD\rho}{\mu_f}$ ,

where u is the inlet velocity, $\mu_f$ is the dynamic viscosity, ρ is the density, and D is the hydraulic diameter. The Reynolds number has been expressed differently throughout the literature. To ensure agreement with the experimental data [1], this study used D=2h.

The model in the COMSOL library has been validated against the experimental data[1]. However, the model is validated using the properties of air. Since liquid metal would be the fluid medium in our study, it is essential to ensure that the model still applies with the properties of this liquid metal, NaK.

With the same step size and the Reynolds number, the liquid metal flow is able to regenerate the same separation and reattachment point as the ones generated by using the properties of air. This also means that the separation and reattachment points in a back-step depend only on the Reynolds number and the step size. To generate the same Reynolds number, a greater velocity is needed since viscosity and density of the liquid metal are different from those of air.

Computed results for Re= 389 show that there is only one recirculating region downstream of the step, (Zone A above). As the Reynolds number increases to Re=648, a second recirculating region appears downstream of the step, at the upper wall (Zone B).

This is consistent with the results obtained using the air properties as well as the experimental results. Thus, the back-step flow without MHD effect is validated.

### 3.3 The MHD Effect on a Back-step Flow

To investigate the MHD effect on a back-step flow, the magnetic field validated previously in the Hartmann problem is applied to the step flow. The back-step geometry is kept the same as the one shown in Figure 9. The mesh for the model is shown in figure 10. The number of degrees of freedom is 19722, which was chosen to produce accurate results in a relatively short time

**Figure10: Mesh for the back-step geometry in COMSOL**

To investigate the MHD effect, inlet velocity and all the fluid properties remain the same. The model would be first run without the magnetic effect. The same model is rerun with the magnetic field applied as shown in Figure 11. The magnetic flux on region 2 is generated the same way as the one shown in Figure 4.

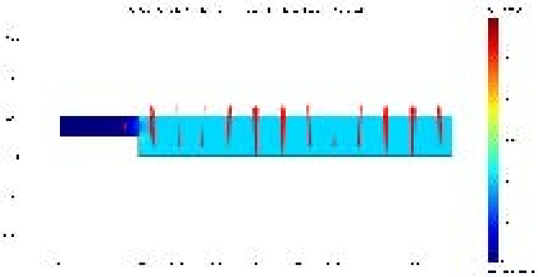

**Figure 11: Application of magnetic field in region 2 (downstream the back-step)**

The magnetic field is applied in the Y-direction only on the 2nd region (beginning at the step) where the velocity profile is of the interest. No magnetic field is applied to the 1st region because the inlet velocity profile needs to be consistent to ensure an accurate comparison. A larger pressure drop is required to maintain a the prescribed inlet velocity profile under the magnetic and electric field. Figure 12 shows the pressure drop in the

2nd region for the model with and without the MHD effect.

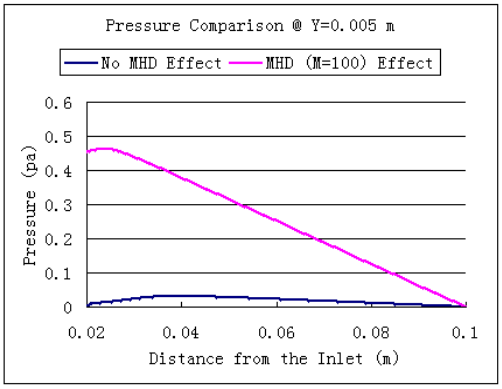

**Figure 12: MHD effect on the pressure drop in the back-step Flow**

For a Re = 100 with M = 0 , the velocity profile in the back-step geometry has one recirculation region downstream of the step (Zone A) , followed by reattachment so that, at the exit, the velocity profile becomes parabolic again. as shown in Figure 13.

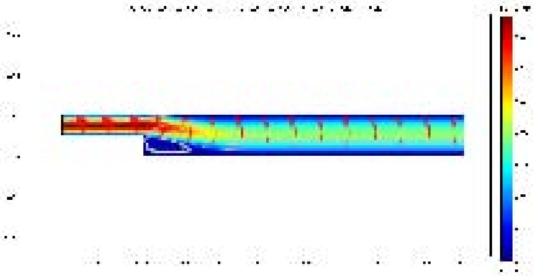

**Figure 13: Velocity field in the back-step geometry with Re=100 and M = 0**

With an applied magnetic field in such that M=100, the recirculation region downstream the step becomes much smaller as shown in Figure 14.

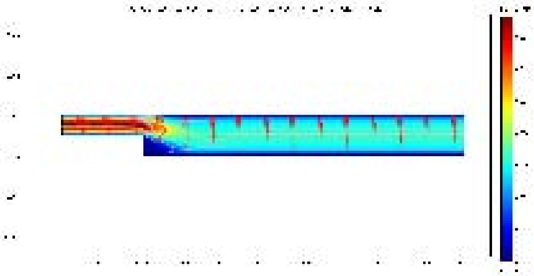

**Figure 14: Velocity field with M = 100**

The velocity profile at the exit exhibits the behavior of the normal flow through the channel under the effect of MHD. The velocity at the exit becomes flatter and the velocity gradient becomes greater near the wall. The detailed effect on the flow pattern around the step varies depending on the strength of the electric and magnetic fields.

A similar effect is observed in the step flow with a higher Reynolds number. At Reynolds number of 648, the second recirculation downstream the step (Zone B) vanishes with a Hartmann number of 100 and the profile looks like the one with a lower Reynolds number.

After the disappearance of the second recirculation region, the velocity profile looks similar to the velocity profile with the smaller Reynolds number. This is anticipated because the implementation of the MHD affects the pressure distribution as well. Separation is intimately connected with the pressure distribution[8]. With M=100 and Re=648, the pressure distribution is more uniform vertically (channel height) at various channel length compared to when M=0 and Re=648.

At the recirculation region, the pressure distribution along the y axis is not uniform as shown in Figures 15 and 16. In the region where recirculation exists for both M=0, and M=100 at X=0.02 through X=0.04, the pressure changes with the channel height. However, with the disappearance of the second recirculation region @ M=100 (Figure 16), the pressure at X=0.06 to X=0.09 along the y axis is much more uniform compared to the case when M=0 (Figure 15).

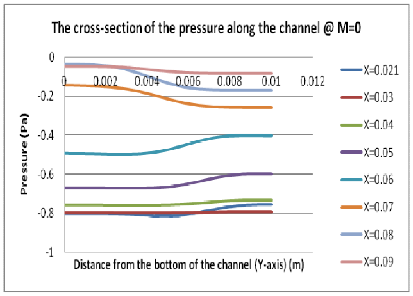

**Figure 15: The cross-section of the pressure along the channel @ M=0**

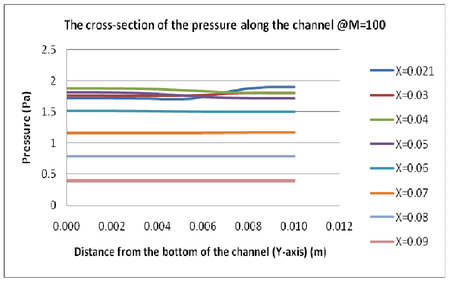

**Figure 16: The cross-section of the pressure along the channel @ M=100**

The disappearance of the second recirculation region in the model is correlated with the change in the pressure. Figure 17 shows that the pressure gradient along the top of the channel for M=100 and M=0. For the regions where recirculation exists, the absolute pressure gradient is much smaller relatively to the region of no recirculation. In addition, the pressure gradient for M=0 exhibits more points of inflexion, which could be the cause of separation.

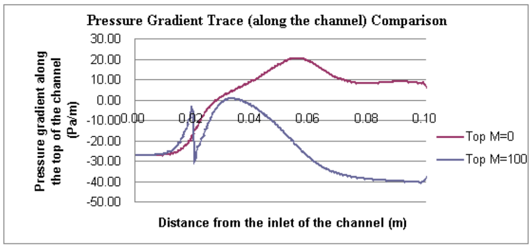

**Figure 17: Pressure gradient trace (along the channel) comparison for M=0 & M=100**

It is expected that all the recirculation regions would disappear given a large enough magnetic and electric fields because the Lorentz force would hinder the flow and change the pressure distribution.

### 3.4. MHD Effect on Heat Transfer in a Backward Step Flow

Since the magnetic and electric fields have significant effect on the velocity profile on the step flow, the heat transfer mechanism in a backward step flow is likely to be affected as well. The heat transfer is modeled in the conduction and convection module with these two mechanisms as the main sources of heat transport. In our study, the inlet is modeled to have a constant temperature at 350 deg Kelvin. The bottom side of the channel has a constant temperature of 300 deg Kelvin. All the other sides are assumed to be thermal insulated. The velocity obtained using the Magnetostatic and Incompressible Navier-Stokes fluid is input into the heat transfer module. In addition, the liquid metal fluid, NaK, properties such as the thermal conductivity and specific heat are inputted into the sub-domain. Figure 18 shows the schematic setup of the boundary condition for the conduction and convection module.

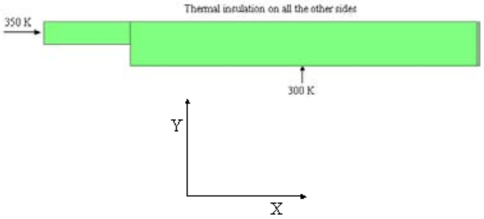

**Figure 18: Boundary condition for the conduction and convection module**

In this case, the effect of convection is small and conduction is the dominant source of heat transfer since the thermal conductivity is large for the liquid metal. The computed temperature fields for the models with and without MHD effect are similar, but there are noticeable differences.

As discussed in Section 3.3, the velocity profile in a back-step configuration changes with the effect of MHD. Since convection is very dependent on the velocity of the flow, the heat transfer is slightly different between the two cases. Figure 19 shows the temperature trace along the topside of the back-step geometry with/without the implementation of the magnetic and electric fields.

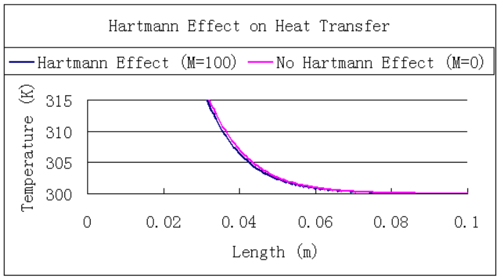

**Figure 19: Hartmann number effect on heat transfer**

The slight difference in the temperature is due to the change in the velocity profile. As presented in Section 3.1, the Hartmann effect generated a greater velocity gradient along the walls and this led to the temperature difference seen along the topside of the geometry.

Much additional information and details about this study, as well as computer programs can be obtained by consulting Mr. Xie's RPI Graduation final report [9].

## 4. Conclusion

The successful validation of the Hartmann problem and the simple backward step flow in COMSOL allows the evaluation of MHD effect in the backward step flow. The result and the analysis indicate that the Lorentz force generated by the magnetic and electric field has a significant effect on the flow pattern in a backward step flow. Just like the simple Hartmann problem between two parallel channels, the Lorentz force generated under MHD in the step flow also flattens the velocity profile and increases the velocity gradients near the wall. Depending on the Hartmann number, the overall velocity profile becomes flatter and smaller in magnitude compared to a parabolic inlet velocity profile shape. This effect on the velocity profile in a backward step flow leads to the change in the separation and reattachment point. Depending on the strength of the fields, the recirculation regions that were once there could become smaller or vanish altogether. This is due to the change in the velocity profile and the pressure distribution in the channel. This change in the velocity profile also alters the heat transfer mechanism in the back-step flow since convection is affected.

## 5. Acknowledgement

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