Numerical Study of Millimeter-Scale Magnetorheological Elastomer Robot for Undulatory Swimming

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CONTENTS

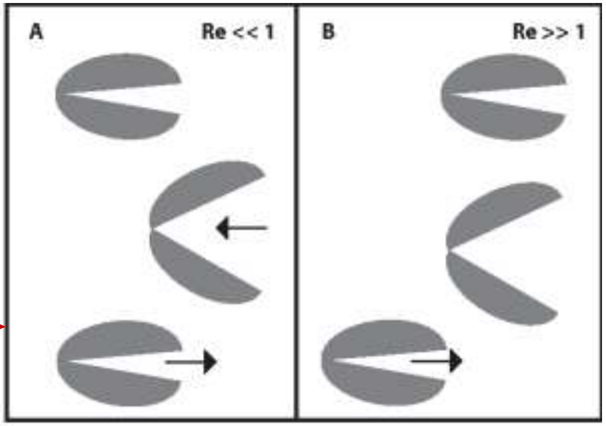
RESEARCH BACKGROUND

Swimming in low-Re regime

Scallop Theorem

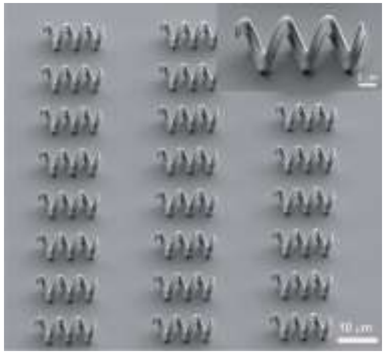
$$Re = \frac{pD_{\mathrm{h}}v_{avg}}{\eta}$$

❌ **None Net Displacement**



Time Unsymmetrical Locomotion

Helical

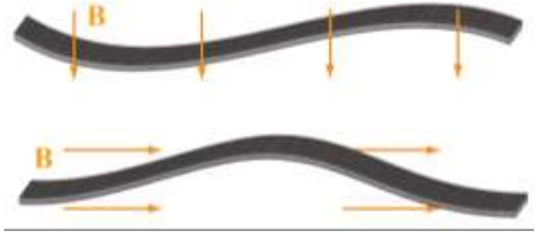


J. Mater. Chem. B **2** (2014) 357–362

Flexible

PloS one **13** (2018) e0206456

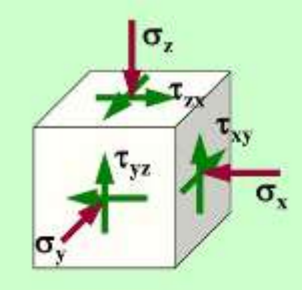
Undulatory



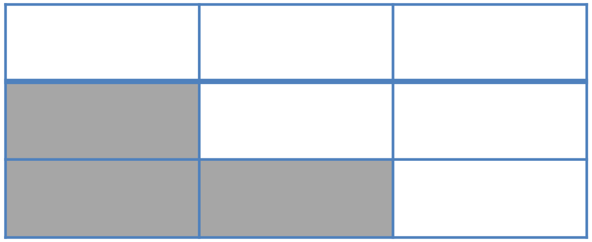
Appl. Phys. Lett. **104** (2014) 174101

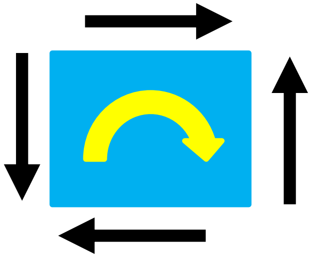
Distributed body torques

Symmetric Cauchy stress



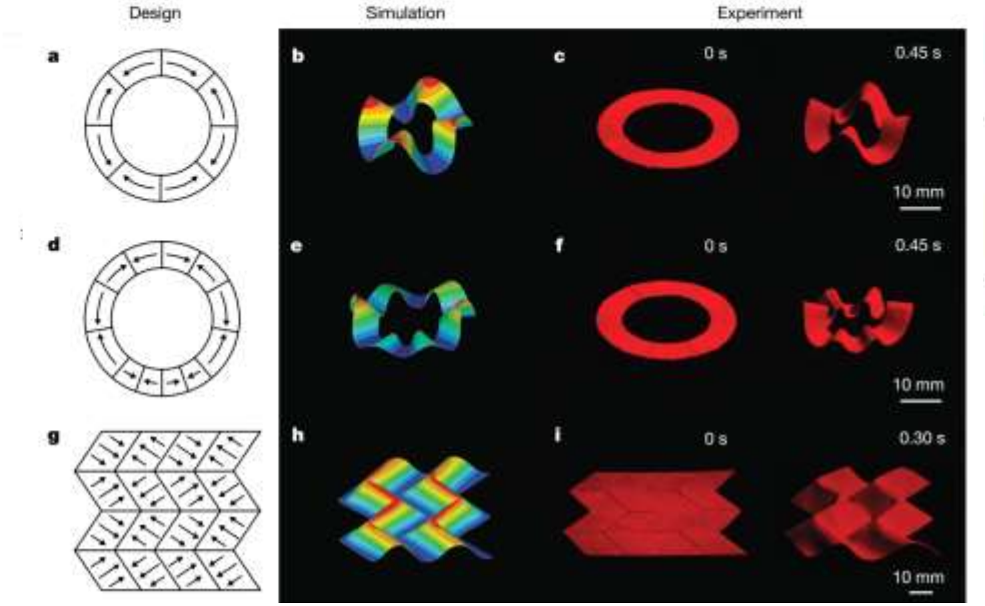
Initial strain





Distributed body torques cause the Cauchy stress to be asymmetric

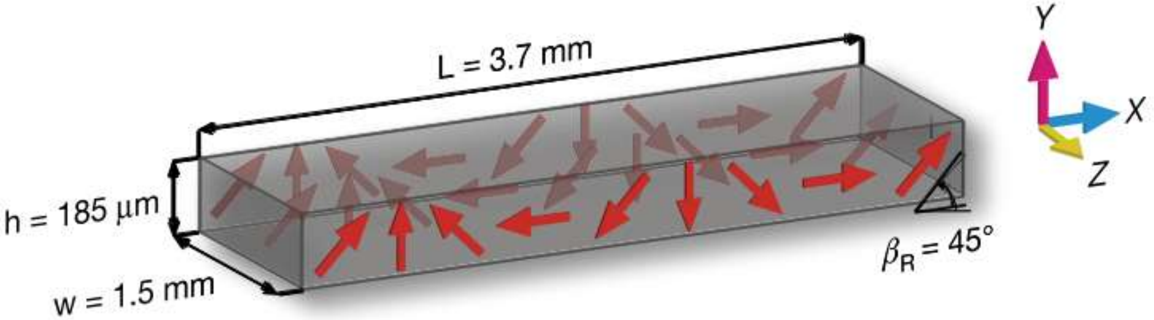
A user-defined element subroutine in Abaqus/Standard



J Mech Phys Solids **124** (2019) 244-263.
Nature **558** (2018) 274.

COMPUTATIONAL METHODS

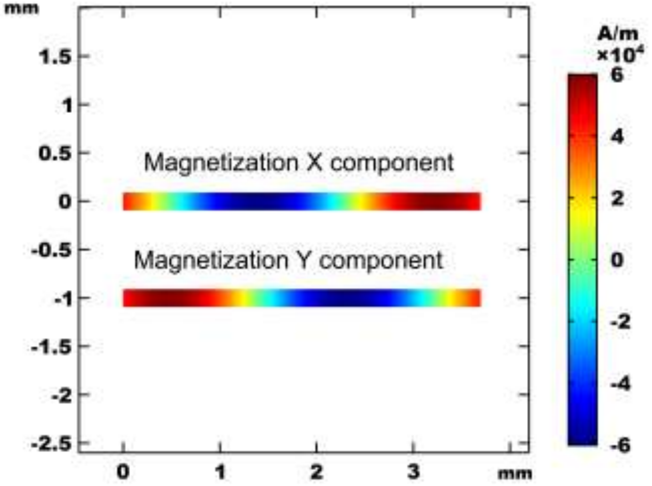
Approximation of distributed torques

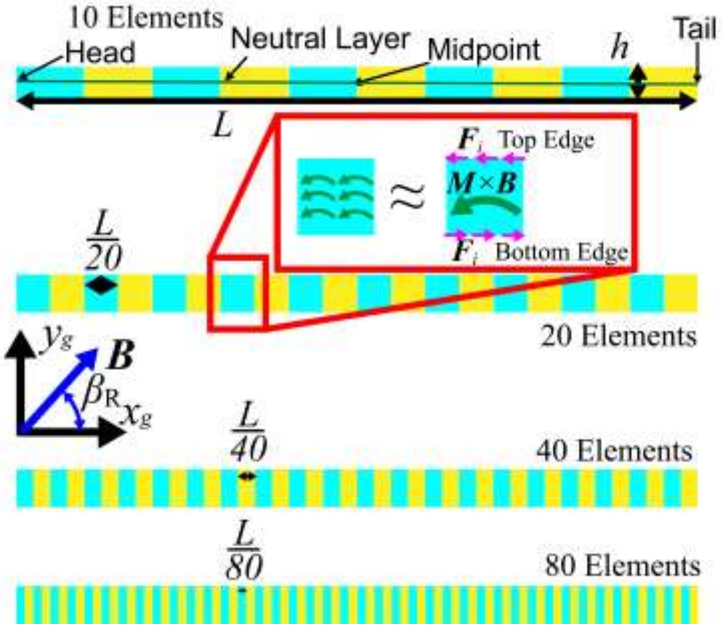


Simple Structure

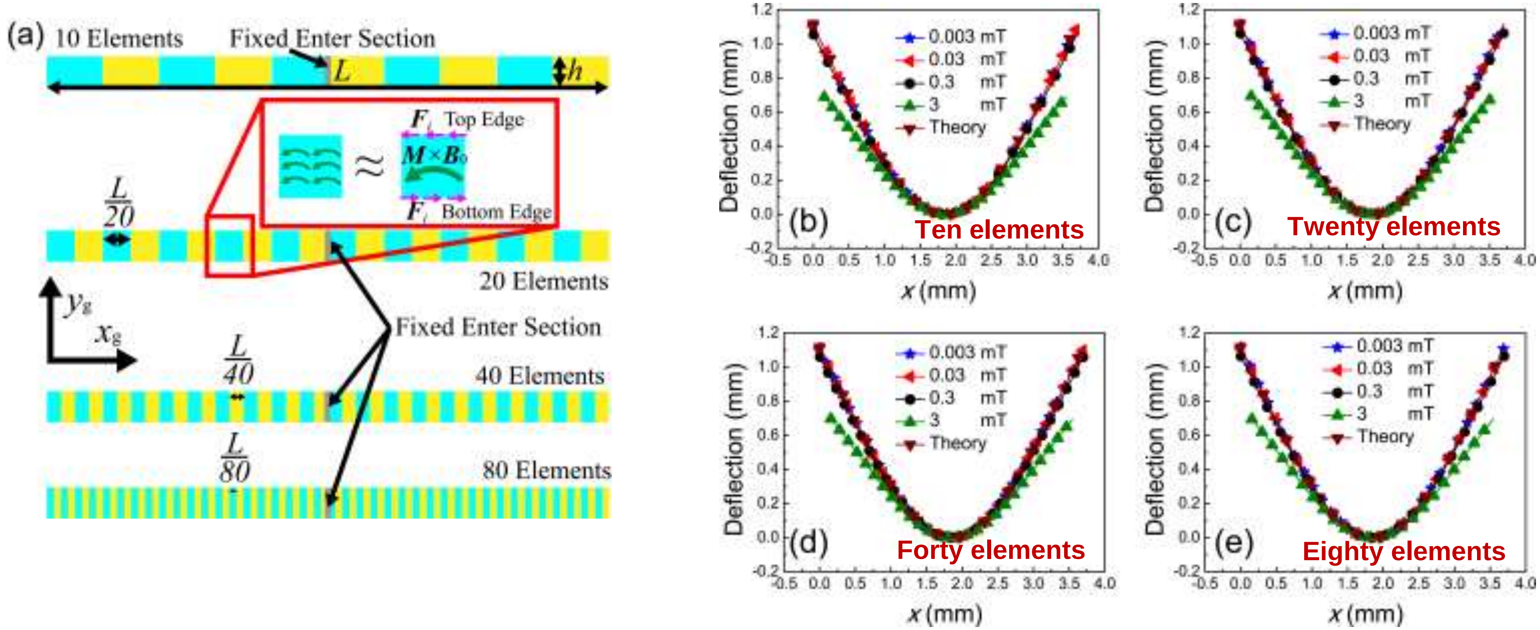
Nature **554** (2018) 81.

- ◆ **2D Approximation**
- ◆ **Approximation of torques**





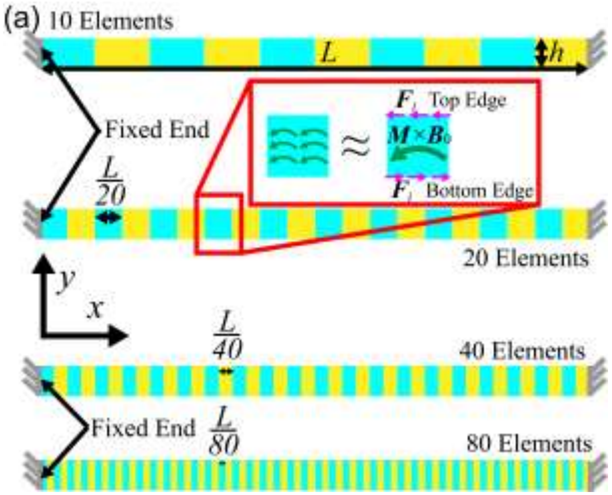
Validation

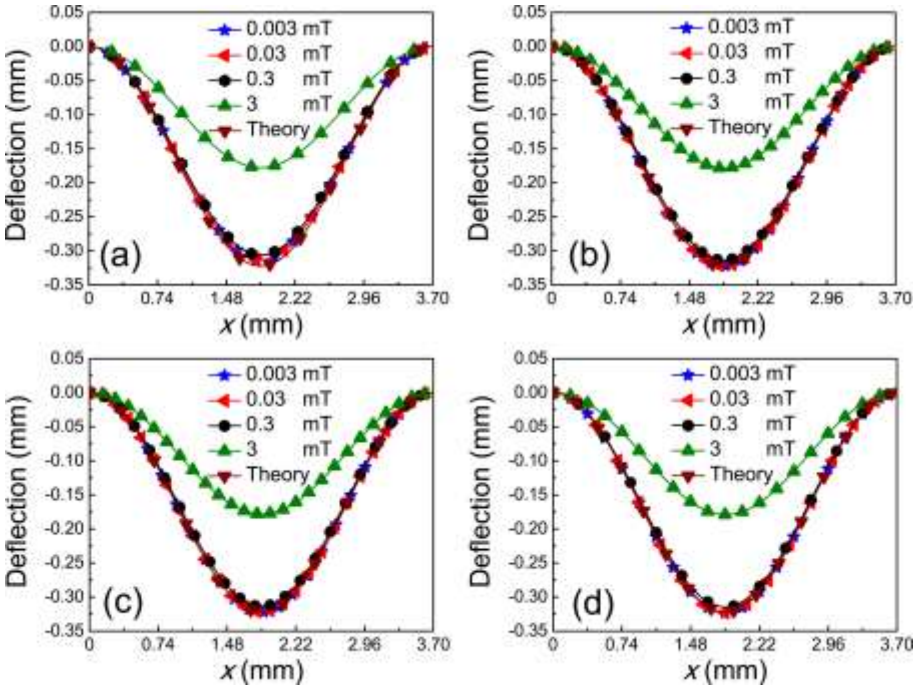


$$y(x,t)=\frac{MAL^3B}{8\pi^3EI}\cos\left(\frac{2\pi}{L}x+\beta_R-2\pi ft\right)+k_1x^2+k_2x+k_3$$

(a) Sketch of the magnetorheological elastomer robots with 3.7 mm in length with the mechanics boundary conditions. Deflections of the robots composed of ten **(b)**, twenty **(c)**, forty **(d)**, and eighty **(e)** elements under free-free end conditions and varying magnetic field flux. The deflections at *B* = 0.003 mT, 0.03 mT, 0.3 mT, and 3 mT, respectively, are multiplied by 1000, 100, 10 and 1, respectively. A theoretical curve under the field strength of 3 mT is plotted for comparison.

Validation

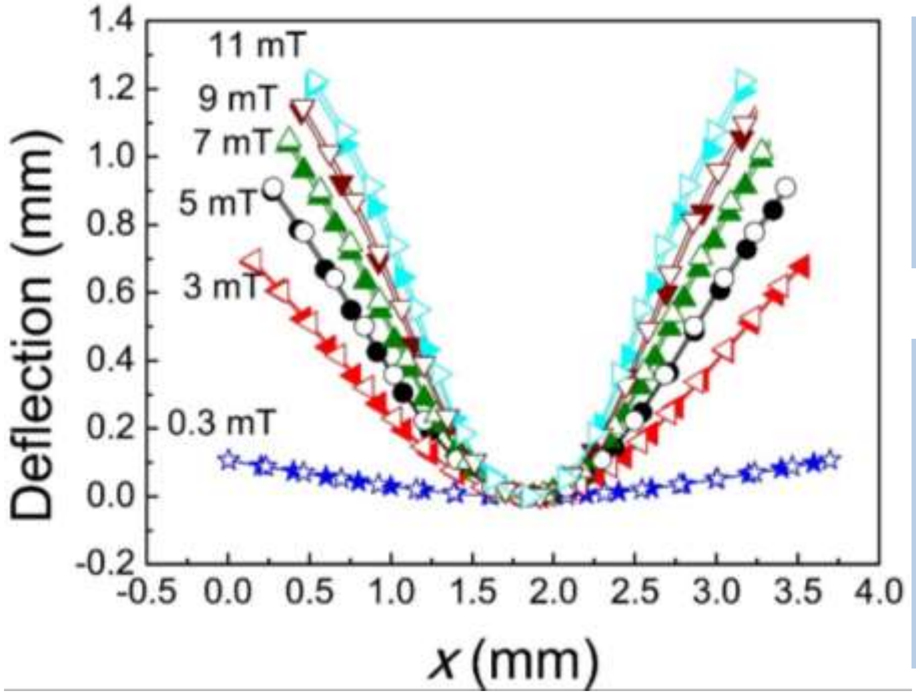




- ◆ **The deformations of the robots are linear when the magnetic field strength is not more than 0.3 mT**

- ◆ **Non-linear deformations of the robots appear under the magnetic field strength of 3 mT**

Validation



Deflection of the robots with ten (solid) and eighty (hollow) elements under free-free end condition and varying magnetic field flux.

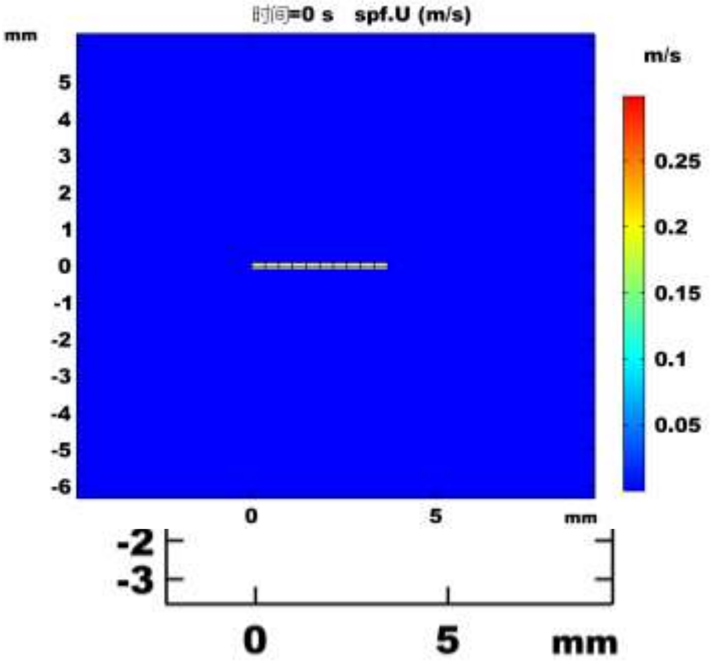
◆ **Increasing number of elements leads to more obvious deflection**

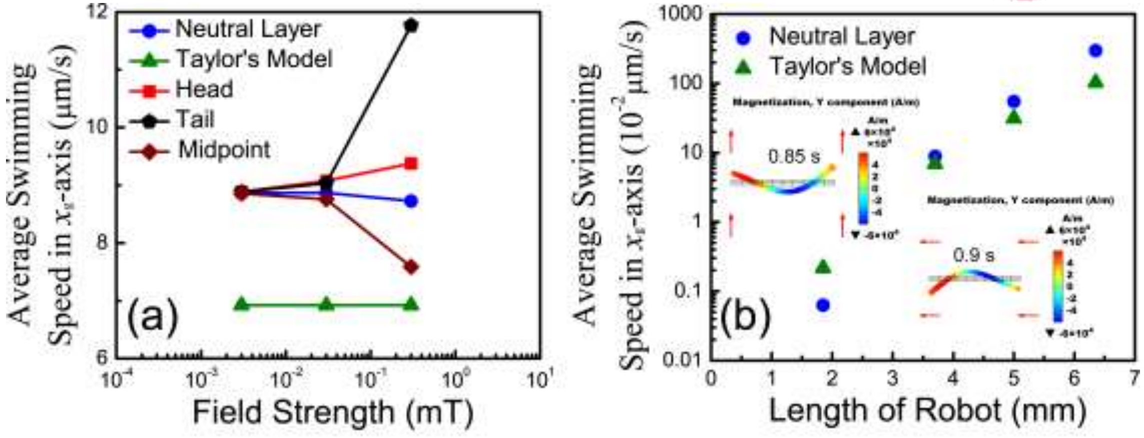
◆ **Below field strength of 3 mT, increasing number of elements plays a feeble role on deflection**

◆ **Above field strength of 7 mT, increasing number of elements obvious impact on deflection**

RESULTS & DISCUSSION

Swimming speed





$$\text{Taylor's model:} \quad V_{\text{swim}} = \frac{2\pi}{L}\left(\frac{3ML^3B}{2\pi^3Eh^2}\right)^2 f$$

Average swimming speed of the ten-element robot of *L* = 3.7 mm under varied magnetic field strength **(a)** and at B = 0.03 mT with various lengths **(b)**. The rotating frequency is 5 Hz. The results obtained from 0.003 mT and 0.03 mT times 10000 and 100, respectively, for clear display.

- **Robot curls and rolls easily in the underwater case, resulting in deteriorated swimming performance**

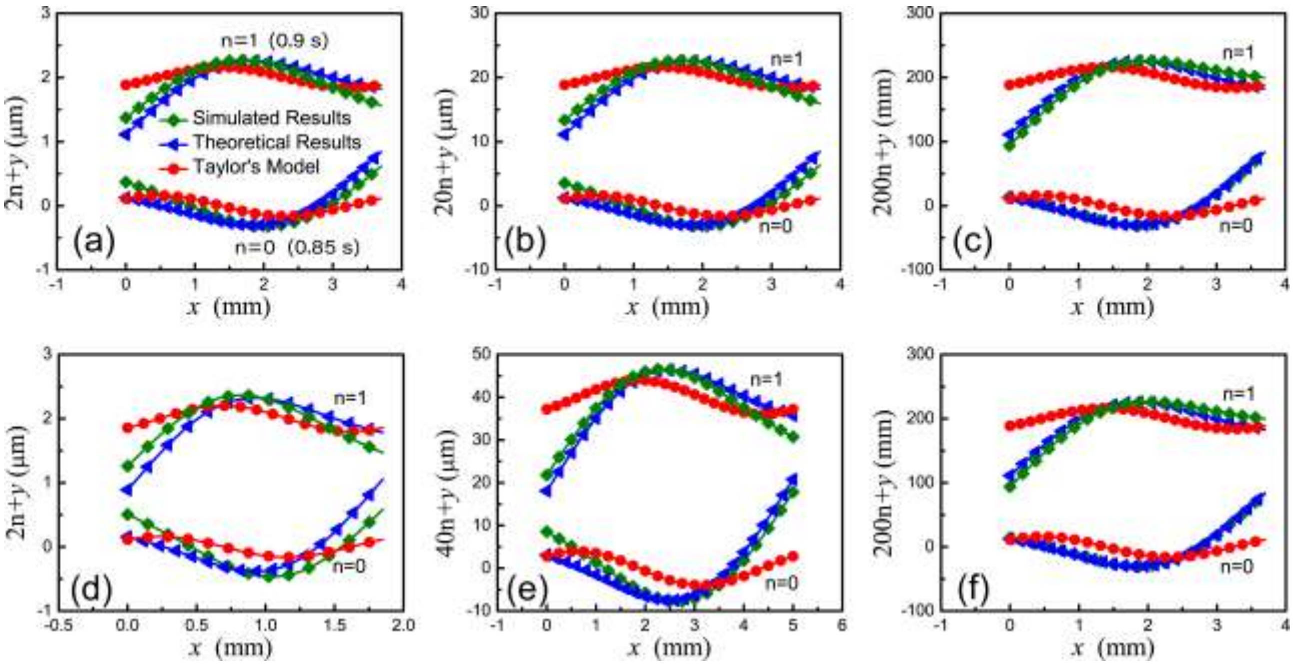
Soft Robot. **5** (2018) 761-776.

Re

L (mm)	B (mT)	Re
3.7	0.003	3.28×10^{-6}
3.7	0.03	3.28×10^{-4}
3.7	0.3	3.23×10^{-2}
1.85	0.03	2.33×10^{-6}
5	0.03	2.74×10^{-3}
6.35	0.03	1.9×10^{-2}

- **Robots in all cases could be treated as low Re swimmers**

Swimming pattern

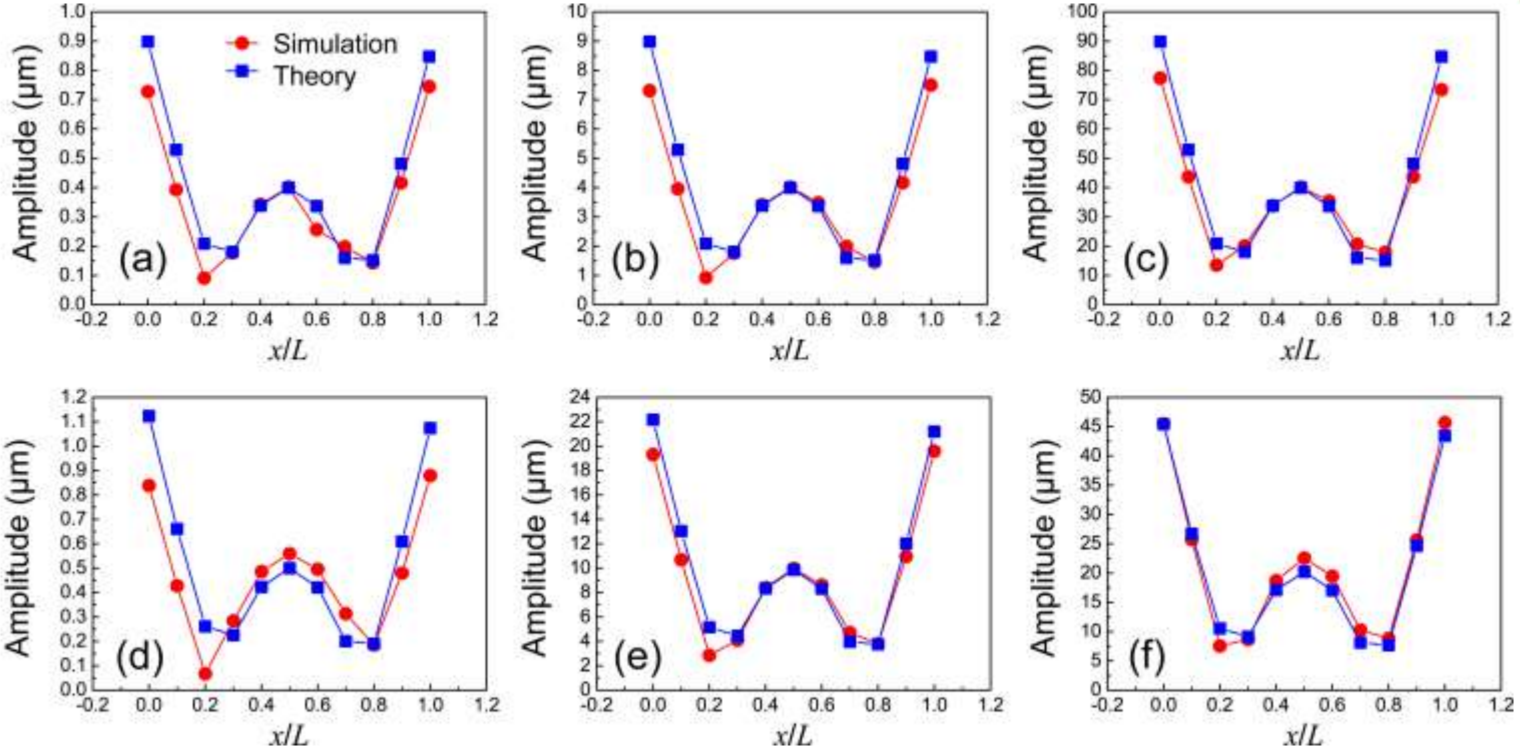


Swimming gaits of the robot of 3.7 mm (a, b, c), 1.85 mm (d), 5 mm (e), and 6.35 mm (f) in length at 0.85 s (n=0) and 0.90 s (n=1) under the field strength of 0.003 mT (a), 0.03 mT (b,d,e,f), and 0.3 mT (c) with a rotating frequency of 5 Hz.

$$y(x,t) = \frac{MAL^3B}{8\pi^3EI}\left[\cos\left(\frac{2\pi}{L}x+\beta_R-2\pi ft\right)+\frac{1}{2}\cos(\beta_R-2\pi ft)\left(\frac{2\pi}{L}x\right)^2-\pi^2\cos(\beta_R-2\pi ft)\left(\frac{2\pi}{L}x\right)-\sin(\beta_R-2\pi ft)\left(\frac{2\pi}{L}x\right)-\frac{241}{50}\sin\left(2\pi ft-\frac{101\pi}{200}\right)\right]$$

$$y(x,t)=\frac{MAL^3B}{8\pi^3EI}\cos\left(\frac{2\pi}{L}x+\beta_R-2\pi f t\right)+k_1x^2+k_2x+k_3$$

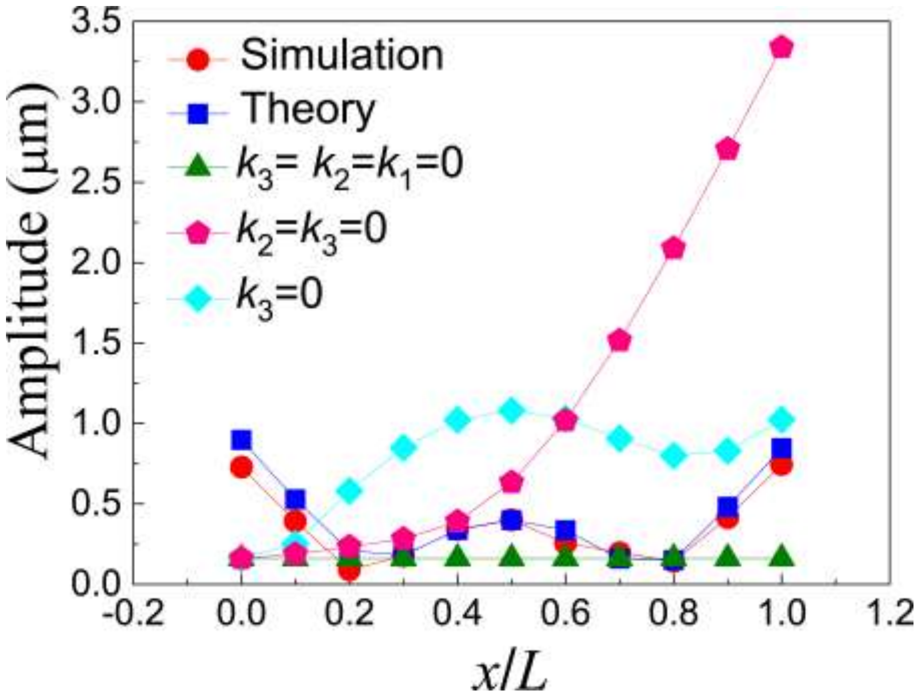
Amplitude



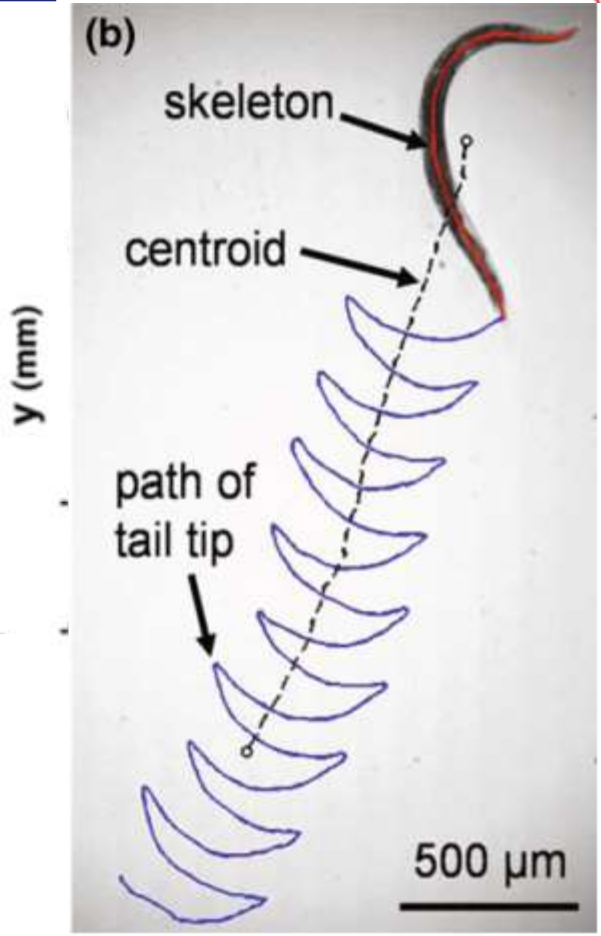
Amplitudes of the points along the neutral layer of the robot of 3.7 mm (a, b, c), 1.85 mm (d), 5 mm (e), and 6.35 mm (f) in length under the field strength of 0.003 mT (a), 0.03 mT (b, d, e, f), and 0.3 mT (c) with a rotating frequency of 5 Hz.

- **Amplitudes are inconsistent**
- **Largest beating amplitudes appear at the head and tail of the robot in both simulations and theories**

Amplitude



Amplitudes of the points along the neutral layer of the robot with 3.7 mm in length predicted by varied values of k_1, k_2, and k_3 under the field strength of 0.003 mT. The simulated results are plotted for comparison.

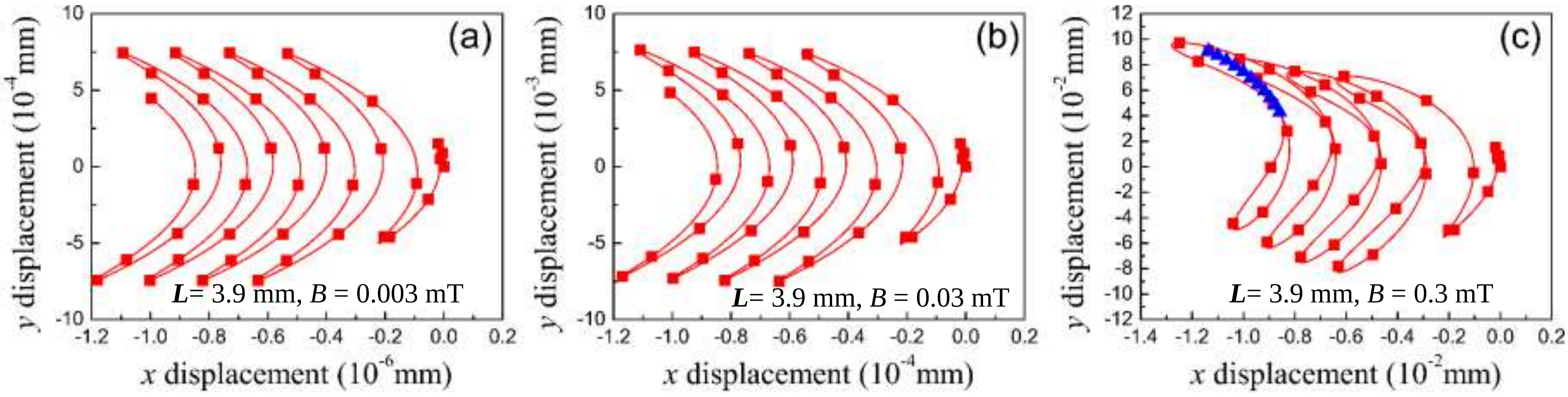


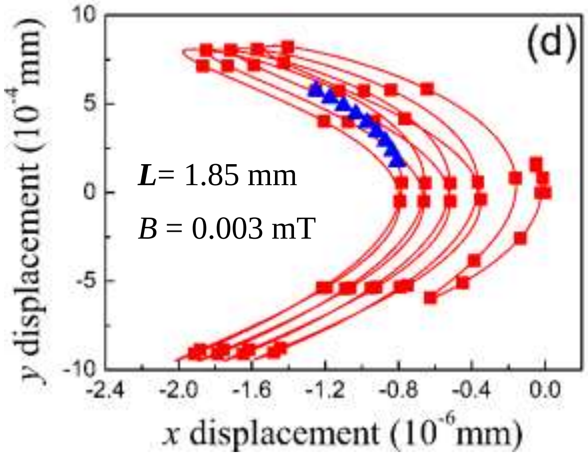
Caenorhabditis elegans

Phys. Rev. Lett. **106** (2011) 208101

◆ **Inconsistency also extensively exists in undulatory microbial swimmers with finite body lengths**

Trajectory of the tail





The trajectory of the tail in the robots with the length of 3.9 mm (a,b,c) and 1.85 mm (d) under the field strength of 0.003 mT (a,d), 0.03 mT (b), and 0.3 mT (c)

- **Countermovement is more pronounced with higher field strength and shorter robot**

CONCLUSION

Conclusion

1. Present an approximation method for the calculation of distributed magnetic torques in soft robot.

2. Demonstrate the invalidation of Taylor’s model in the soft robot with continuously magnetization profile .

3. Present a novel swimming gait function for the soft robot with continuously distributed magnetization profile.

Thanks for your attention!