

Numerical Evaluation on Cutoff Frequency of Twisted Waveguide with Rectangular Cross-section

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Abstract: In this study, the cutoff frequency of twisted waveguide with rectangular cross-section is numerically evaluated by COMSOL Multiphysics 5.4. Results of simulation agree with theoretical result. Propagation of microwave in this type of waveguide is investigated. With the aid of application builder, efficient design of twisted waveguide with rectangular cross-section are achieved.

Keywords: numerical evaluation, twisted waveguide, cutoff frequency, design based on application builder

1. Introduction

Waveguide can be regarded as a special form of transmission line which consists of a hollow metal tube and works for propagating signals of extremely high frequency. The wavelength approaches the cross-sectional dimensions of the waveguide. Practically, misalignment, where two connecting flanges are offset and cannot be fitted or bolted together, is an issue for utilization of waveguide. Therefore, it is important to apply bended or twisted waveguide for these troubled situation [1]. Furthermore, it is meaningful to consider usage of flexible waveguide for kinds of applications, such as the fifth generation mobile communication.

This work performs a simulation study of twisted waveguide with rectangular cross-section using COMSOL Multiphysics 5.4. The cutoff frequency of this type of waveguide is evaluated firstly. Then the propagation of microwave is discussed in the twisted waveguide. One Application is built based on these simulations by application builder for efficient design of twisted waveguide with rectangular cross-section.

2. Theoretical Analysis

A theoretical analysis of propagation constants in twisted rectangular waveguides has already been introduced by Lewin[2]. Subsequent research studied dispersion formulas for twisted rectangular waveguides by Hatsuo et al[3]. According to their results, the cutoff frequency could be calculated based on the equation (1)

$$f_{cr} = \frac{c}{2r_1 a} = \frac{f_c}{r_1} \quad (1)$$

where c is the velocity of light, a is the side of rectangular cross section, f_c is the cutoff frequency of the straight waveguide with the same cross section. Factor r_1 is decided by the equation (2) and presented graphically in figure 1 of reference [2]. The factor decreases with increasing $\bar{\alpha}$. Here, $\bar{\alpha} = \alpha a / 2\pi$ denotes the degree of twist as a dimensionless quantity and twist constant α (rad/m) is assumed to be small.

$$r_1 = \sqrt{\frac{1 - A_2'(\varnothing)\bar{\alpha}^2}{1 - A_2''(\varnothing)\bar{\alpha}^2}} \quad (2)$$

in which

$$A_2'(\varnothing) = \frac{8}{3}\varnothing^2 - 6 - \pi^2 + \frac{128}{\pi^2} \cdot \frac{\tan\varnothing}{\varnothing} + \frac{256}{\pi^2\varnothing} \cdot \sum_{m=2,4,\dots}^{\infty} \frac{(m^2 + 1)^2 \tanh(\varnothing\sqrt{m^2 - 1})}{(\sqrt{m^2 - 1})^9} \quad (3a)$$

$$A_2''(\varnothing) = -\frac{4}{3}\varnothing^2 + 6 + \pi^2 - \frac{128}{\pi^2} \cdot \frac{\tan\varnothing}{\varnothing} - \frac{256}{\pi^2\varnothing} \cdot \sum_{m=2,4,\dots}^{\infty} \frac{(3m^2 + 1)\tanh(\varnothing\sqrt{m^2 - 1})}{(\sqrt{m^2 - 1})^9} \quad (3b)$$

Where $\varnothing = \pi b / 2a$, and b/a is referred to as the waveguide aspect ratio.

3. Simulation Set-up

The simulation includes two calculations. First calculation is performed to evaluate the cut off frequency of twisted waveguide and second one is carried out to demonstrate the wave propagation in twisted waveguide. The electromagnetic waves, frequency domain interface of RF module is selected for this modeling. One hollow twisted waveguide with rectangle cross-section 40 mm × 20 mm is defined. Besides, the length of the waveguide is set to 100 mm. The twist angle is designed to vary from 0 degree to 90 degree with interval of 30 degree for evaluating the effect of twist angle on cutoff frequency. The waveguide walls are considered to be made of a perfect conductor, therefore boundary condition of perfect electric conductor is adopted.

Eigen frequency study is adopted for the first calculation. To get the cut off frequency, two sides which means the inlet and outlet of twisted waveguide,

are determined by boundary condition of perfect magnetic conductor, as fig.1 shows.

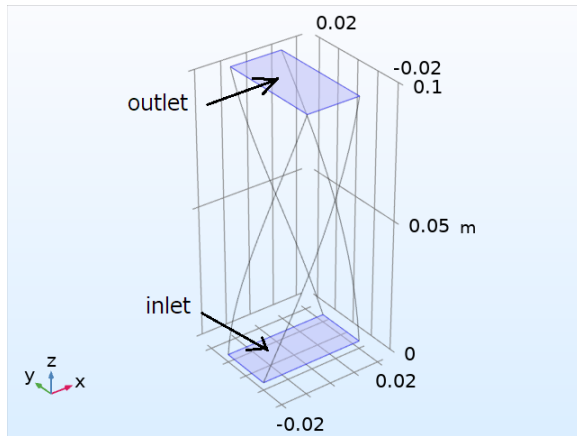


Figure 1. Geometry of twisted rectangular waveguide.

During the second calculation, study of frequency domain is utilized. Different from the first one, boundary condition of port is applied for inlet and outlet of the twisted waveguide. The TE₁₀ mode is excited on at the inlet and off at the outlet, which means no electric field component in the direction of propagation. The solved equation is showing as following,

$$\nabla \times \mu_r^{-1}(\nabla \times \vec{E}) - k_0^2 \left(\epsilon_r - \frac{j\sigma}{\omega\epsilon_0} \right) \vec{E} = 0 \quad (4)$$

Where \vec{E} denotes the electric field, μ_r the relative permeability, σ the electrical conductivity, k_0 the free space wave number, j the imaginary unit. Beside, ϵ_r and ϵ_0 are the relative permittivity and the permittivity of free space, respectively.

Finer free tetrahedral mesh is built and the complete mesh consists of about 7600 domain element. Only several seconds are needed to finish the calculation.

4. Results of Simulation

The S-parameter of waveguide are shown as functions of the frequency in fig. 2. It is observed that a shift of the cut off frequency occurs in case of twisted waveguide. No matter how is the twist angle, only electromagnetic wave with sufficient high frequency could be propagated along the waveguide, which indicates why the waveguide could work as high pass filter.

Figure 3 shows the comparison between simulation result and theoretical result. Results basically do agree with each other. The cut off frequency increases with rise in twist angle. However, the discrepancy between them becomes larger and larger. It is probably caused by the observational error on the reading of figure 1 of reference [3].

Furthermore, figure 4 illustrates the distribution of electric field in twisted rectangular waveguide when twist angle is 90 degree and applied frequency is 7 GHz which is selected so that the TE₁₀ mode is the only propagation mode through the twisted rectangular waveguide. The view is rotated for clear illustration. Then, based on the result, the dynamic wave propagation could be demonstrated with aid of animation feature in post processing, which will be introduced into design of application of this simulation.

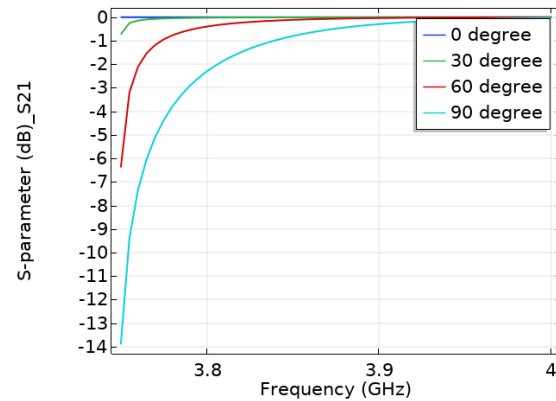


Figure 2. The S-parameters, on a dB scale, as a function of the frequency.

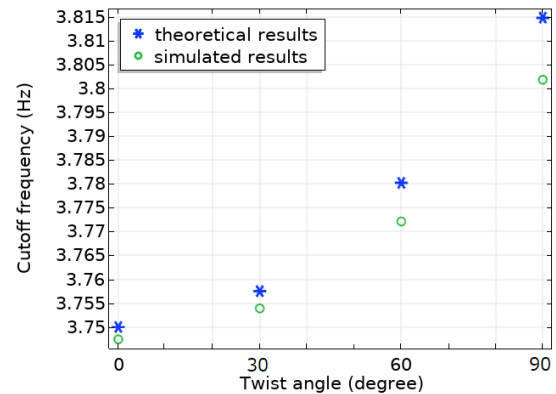


Figure 3. Cutoff frequency versus twist angle. Comparison between theoretical results and simulated results.

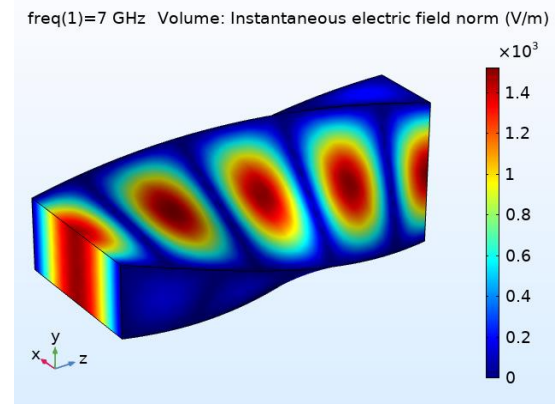


Figure 4. Electric field in twisted rectangular waveguide. Twist angle is 90 and applied frequency is 7 GHz.

5. Design based on Application Builder

One application is designed according to above simulations for efficient determination on twisted rectangular waveguide and demonstration dynamic wave propagation. Figure.5 shows the contents on the first page of the application. Important parameters were required to be input, such as twist angle, length of waveguide, width and height of cross-section. Meanwhile, geometry and mesh were presented at the same page, which are defined by these parameters. On the second page, results of two studies are presented. As fig. 6 shows, first study computes the cutoff frequency and output data into one list. Computation of second study is for demonstrating the wave propagation under certain frequency, which is definitely selected as one higher than cutoff frequency. Obviously, time for modifying geometry and managing results was greatly shortened with help of the optimized application. Therefore, it is meaningful to design the application after making basic modelling.

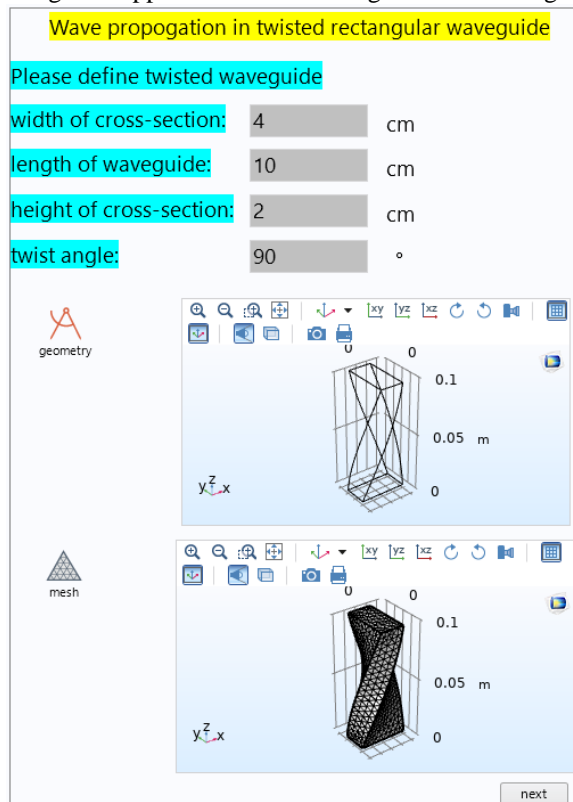


Figure 5. Contents of first page of application.

6. Conclusions

This work investigated the cutoff frequency of twisted waveguide with rectangular cross-section in simulation. Results of simulation agree with theoretical result. Furthermore, propagation of

microwave in this type of waveguide is discussed. With the aid of application builder, efficient design of twisted waveguide with rectangular cross-section are achieved.

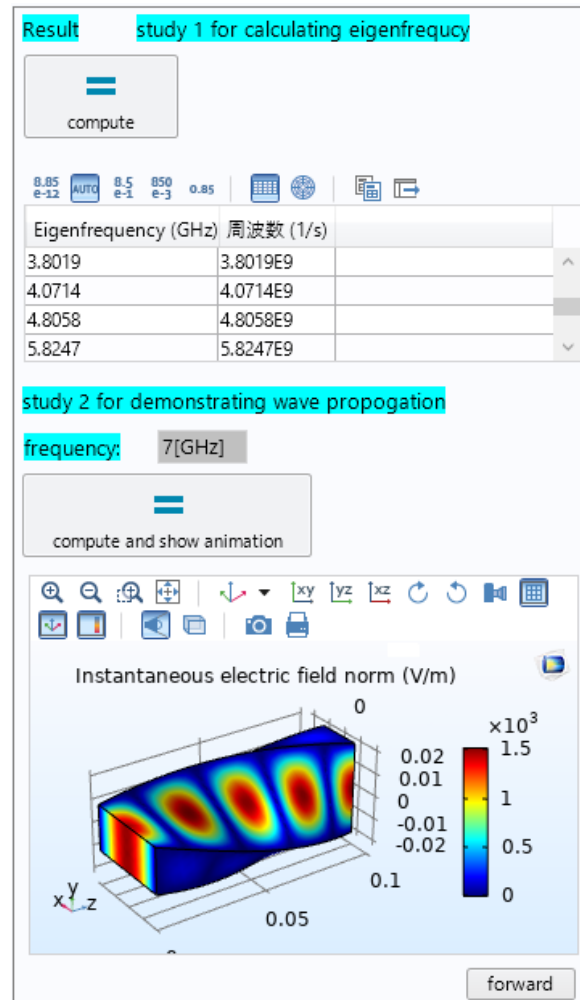


Figure 6. Contents of second page of application.

References

1. M. Sun, et al., Design of a compact and broadband 90-degree waveguide twist. 2018 IEEE International Vacuum Electronics Conference. 24-26 April 2018
2. L. Lewin, Propagation in curved and twisted waveguides of rectangular cross-section, Proc. Inst. Elec. Eng., vol. 102, B, 1, pp. 75-80, 1955.
3. H. Yabe, et al., Dispersion characteristics of twisted rectangular waveguides, IEEE Transactions on microwave theory and techniques, vol. MTT-32, No.1, January 1984.