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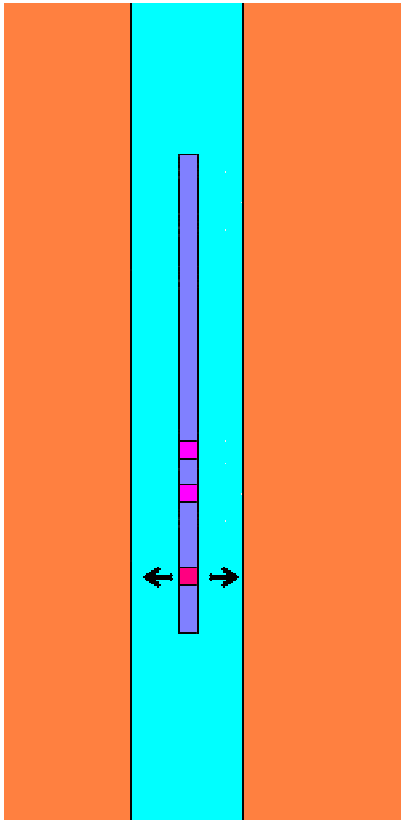
# Numerical Study on Acoustic Field Generated by Dipole Sources in Noncircular Pipe

Hailan Zhang, Weijun Lin, Xiuming Wang

Institute of Acoustics,
Chinese Academy of Sciences
Beijing, China

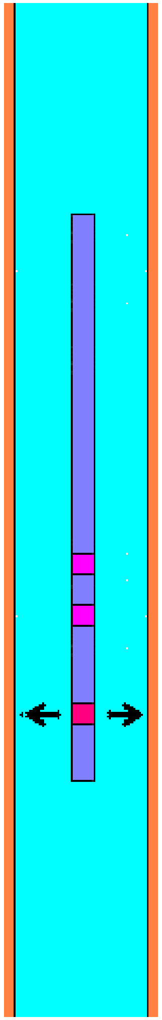
# Background

- Acoustical well logging is an important technology for petroleum industry
- Calibration and testing of tools in real wells is not feasible because the cost is high and the condition is not controllable



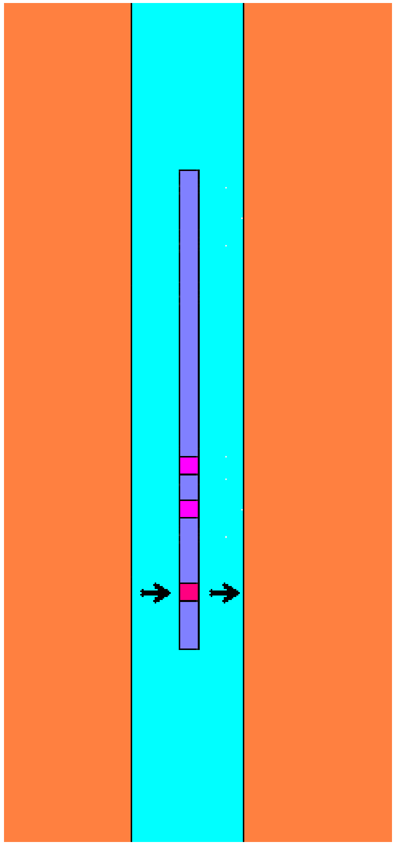
# Background

- During manufacturing and maintaining, the logging tools are usually tested in a fluid filled circular pipe
- Received signals are compared to the calculated results



# Background

- Dipole logging tools are widely used recently.
- Shear wave velocity of formation
- For anisotropic formation two shear wave velocities obtained with different polarization of tool



# Background

- Dipole toold transmit nonaxisymmetric acoustical fields of various polarizaion
- The test result in a circular pipe  is a kind of average over all directions
- Not suitable for nonaxisymmetric dipole tools
- Test in noncircular pipes?

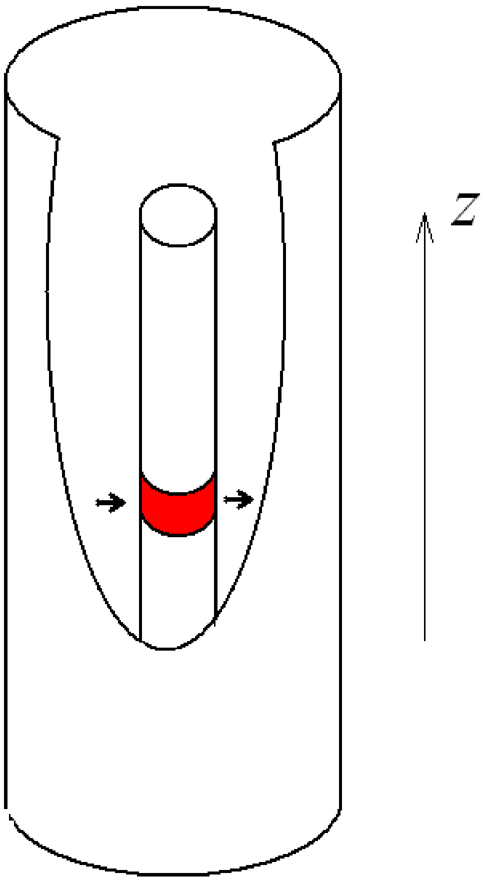
# Problem

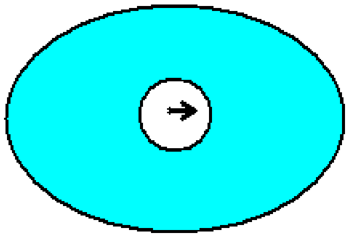
- Liquid

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}-\frac{\partial^2 p}{c^2\partial t^2}=0$$

- Pipe wall $\quad v_n\big|_{\text{outer surface}}=0$
- Tool surface

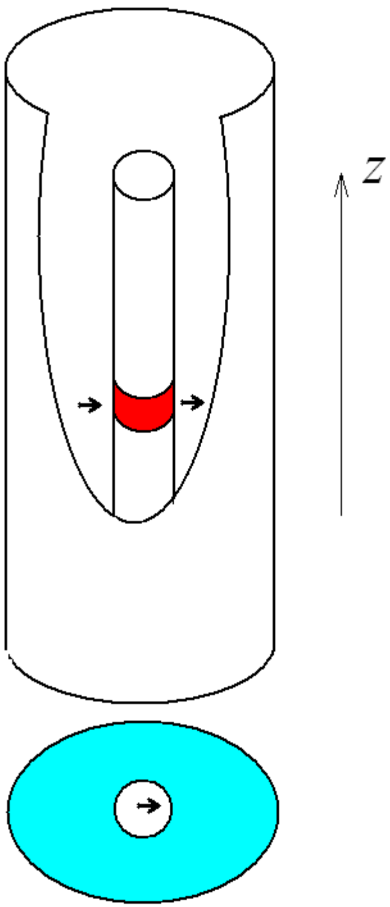
$$v_n\big|_{\text{inner surface}}=v_n(t,z)=\begin{cases} v_0(t)\cos\theta & |z|<d \\ 0 & |z|>d \end{cases}$$



# Calculation Method

- Geometry and medium indenpendent of *z*
- Transformed into Frequency-Wavenumber Domain




$$p(t,x,y,z) = \iint P(\omega,k,x,y)\exp\left[i(kz-\omega t)\right]$$

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \left(\frac{\omega^2}{c^2} - k^2\right)P = 0$$

$$V_n\Big|_{\text{outer surface}} = 0 \qquad V_n\Big|_{\text{inner surface}} = V_n(k,\omega)$$

# 2.5D Method

- Given $\omega$ and $k$, 2D problem, 2.5D method
- Comsol Multiphysics package, PDE mode

$$\nabla \cdot \left( -\boldsymbol{c} : \nabla \boldsymbol{U} - \boldsymbol{\alpha} \boldsymbol{U} + \boldsymbol{\gamma} \right) + \boldsymbol{\beta} \cdot \nabla \boldsymbol{U} + \boldsymbol{a} \boldsymbol{U} = \boldsymbol{f}$$

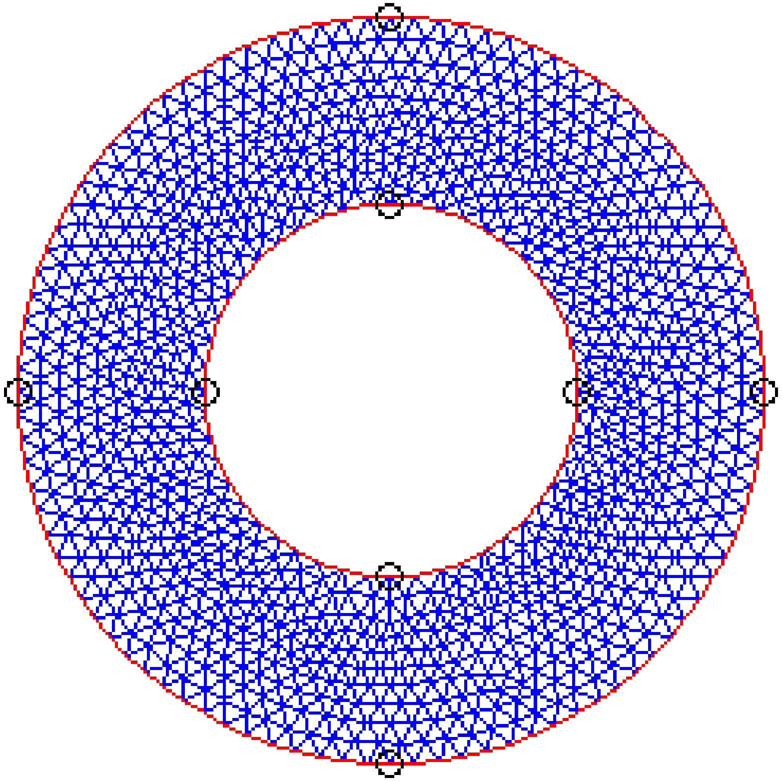
$$\boldsymbol{n} \cdot \left( \boldsymbol{c} : \nabla \boldsymbol{U} + \boldsymbol{\alpha} \boldsymbol{U} - \boldsymbol{\gamma} \right) + \boldsymbol{q} \boldsymbol{U} = \boldsymbol{g} - \boldsymbol{h} \boldsymbol{\mu}$$

$$\boldsymbol{h} \boldsymbol{U} = \boldsymbol{r}$$

$$c = \lambda \qquad a = \lambda k^2 - \rho \omega^2 \qquad \boldsymbol{\alpha} = \boldsymbol{\beta} = \boldsymbol{\gamma} = f = 0$$

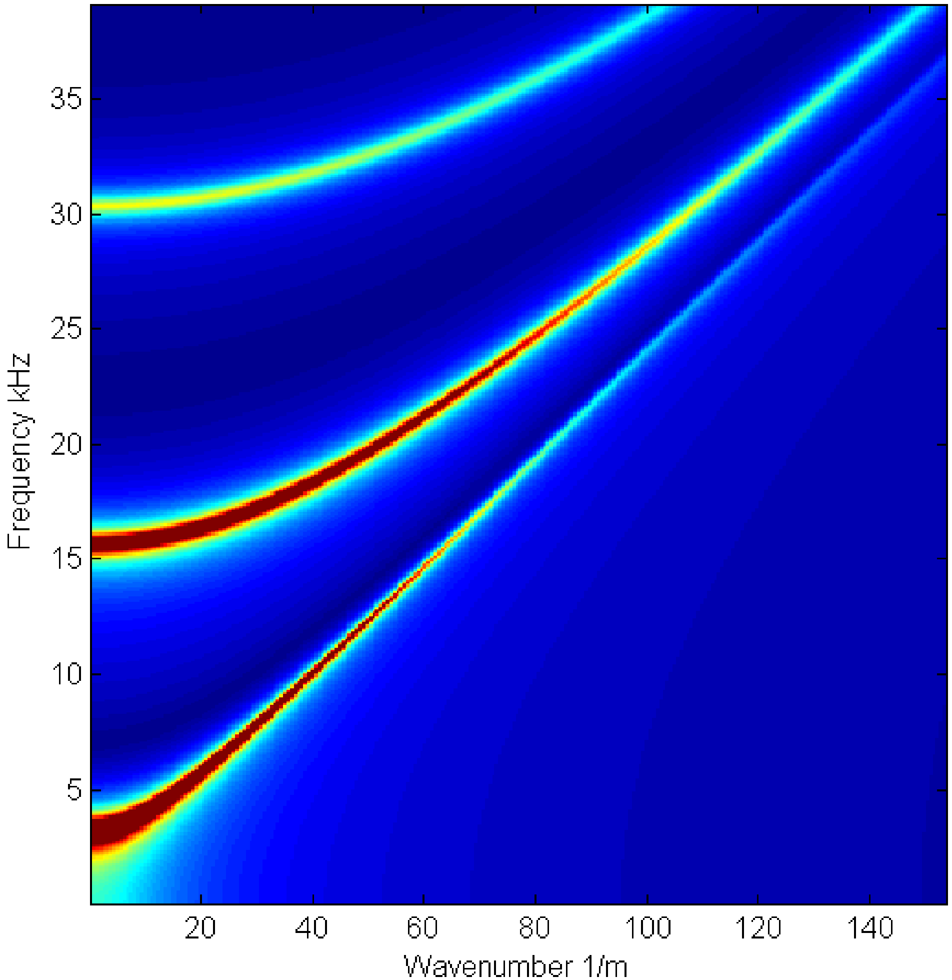
# Circle Pipe

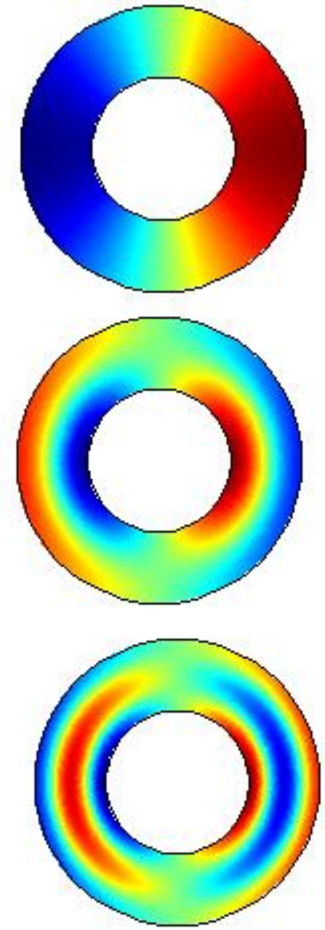
- Diameter 200mm
- Tool Dia. 100mm
- Frequency
  0-40KHz
- Wavenumber
  0-150(1/m)



# Result for Circle Pipe

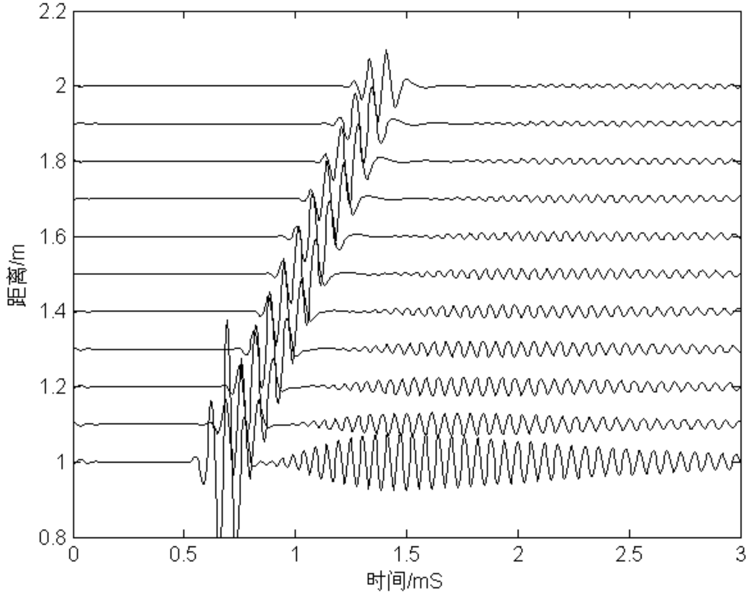
- The result is compared with the analytical solution to verify the method
- The difference is less than 0.1%



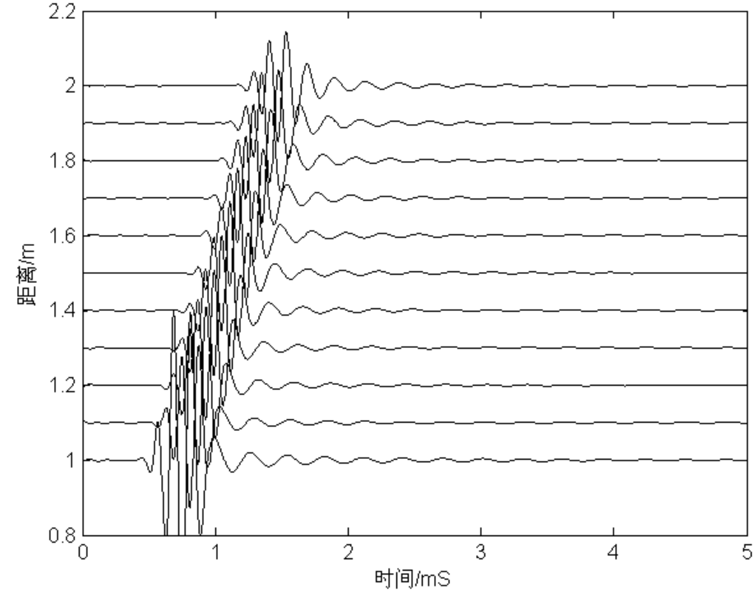



$$v_n\Big|_{\text{inner surface}} = v_n(t,z) = \begin{cases} v_0(t)\cos\theta & |z| < d \\ 0 & |z| > d \end{cases}$$

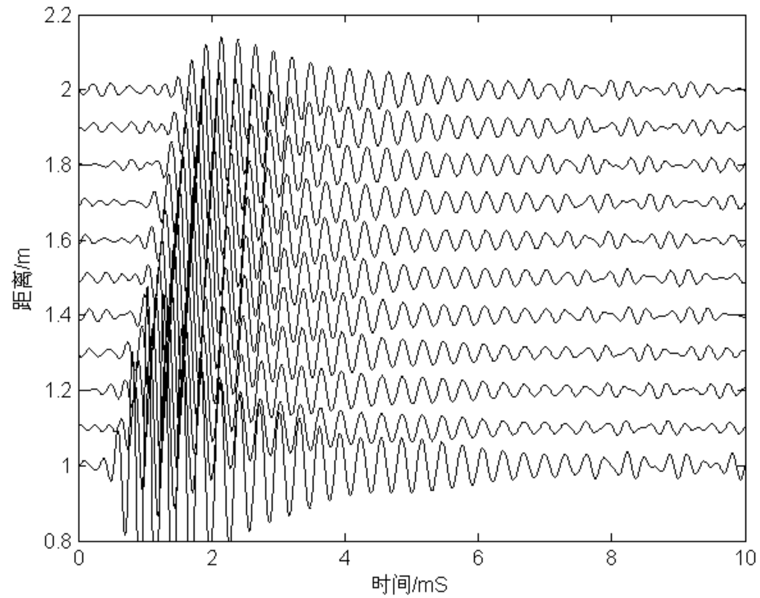
## 14KHz

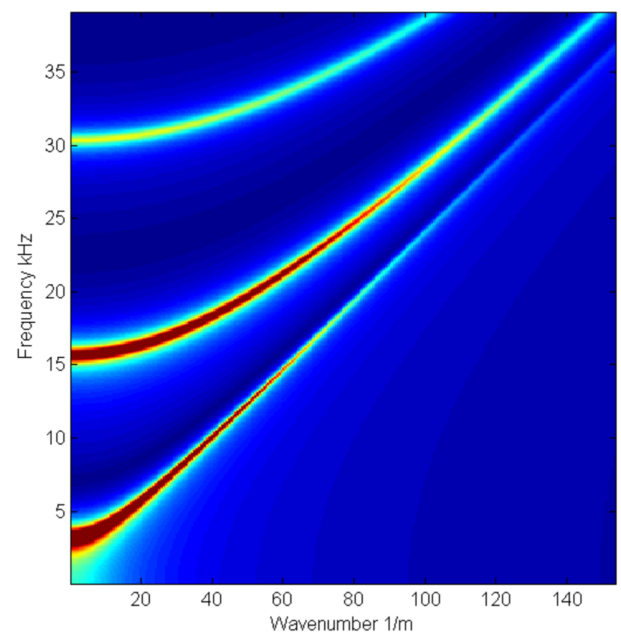


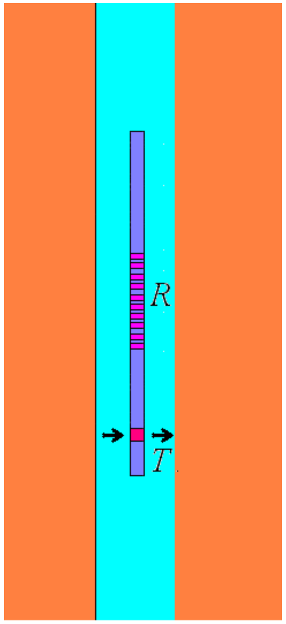

## 8KHz




## 4KHz



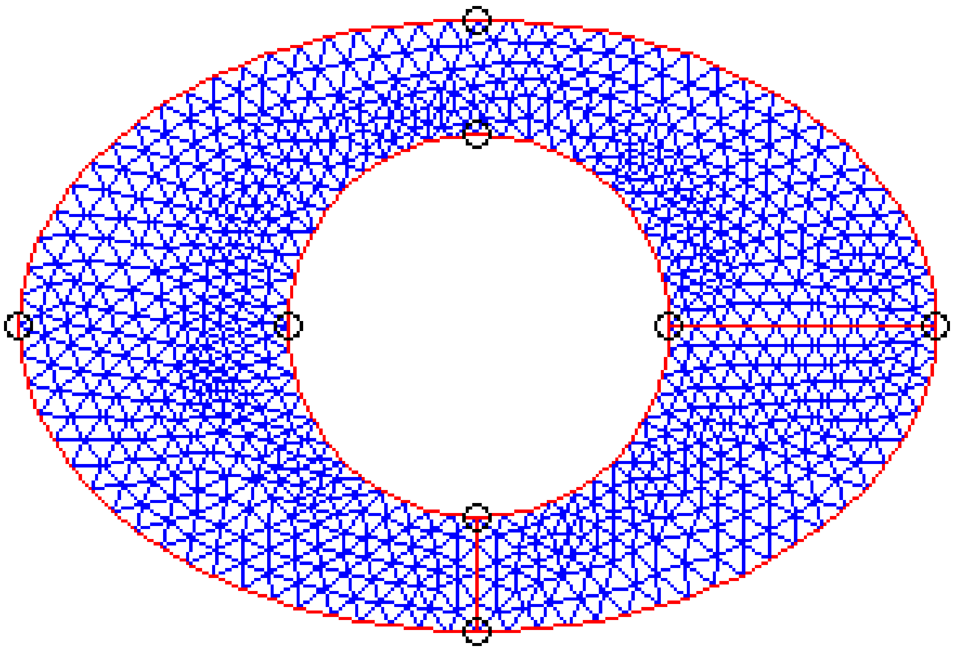
# Advantages of 2.5D Method

- For one point in trasformed domain 2D calculation
- 1 second
- 400 frequencies, 250 wavenumbers
- $\sim$24 hours

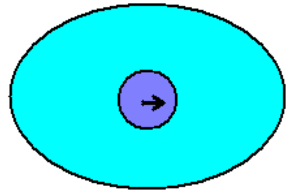
- Parallel calculation
- No artificial ends of the pipe

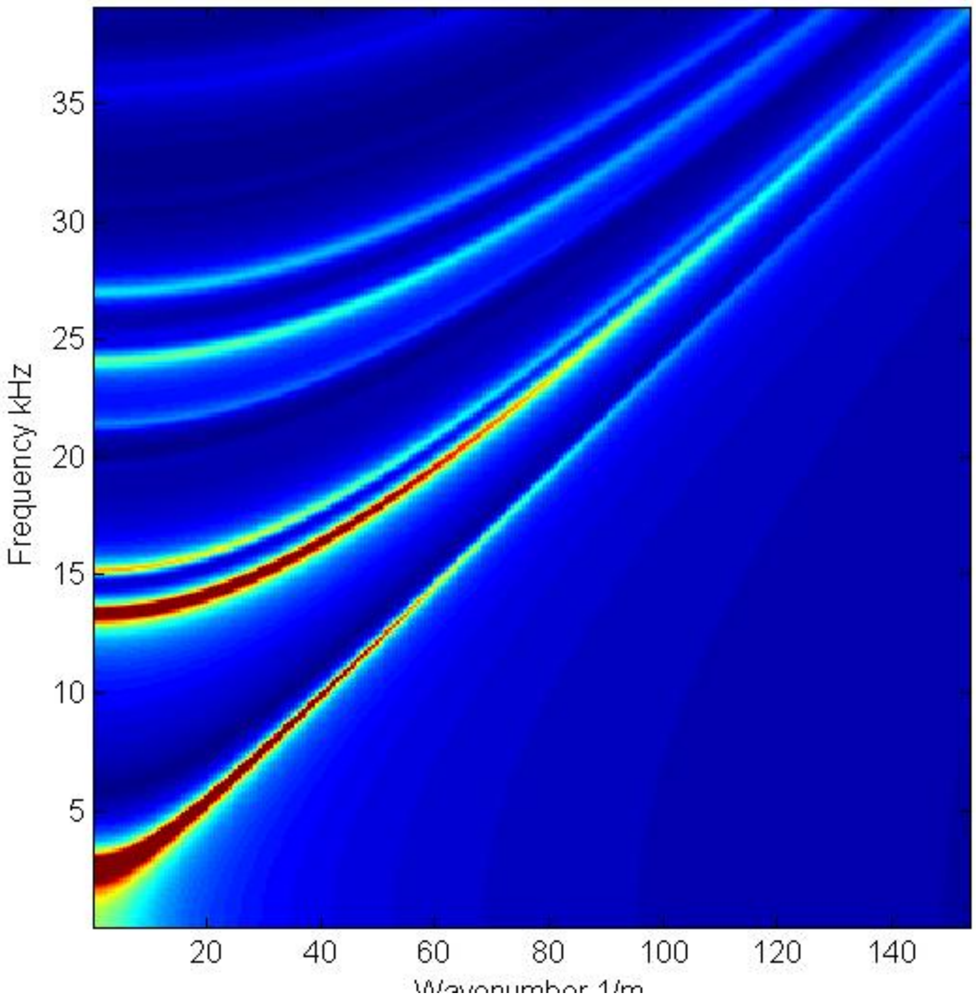
- Limitation

# Elliptical Pipe

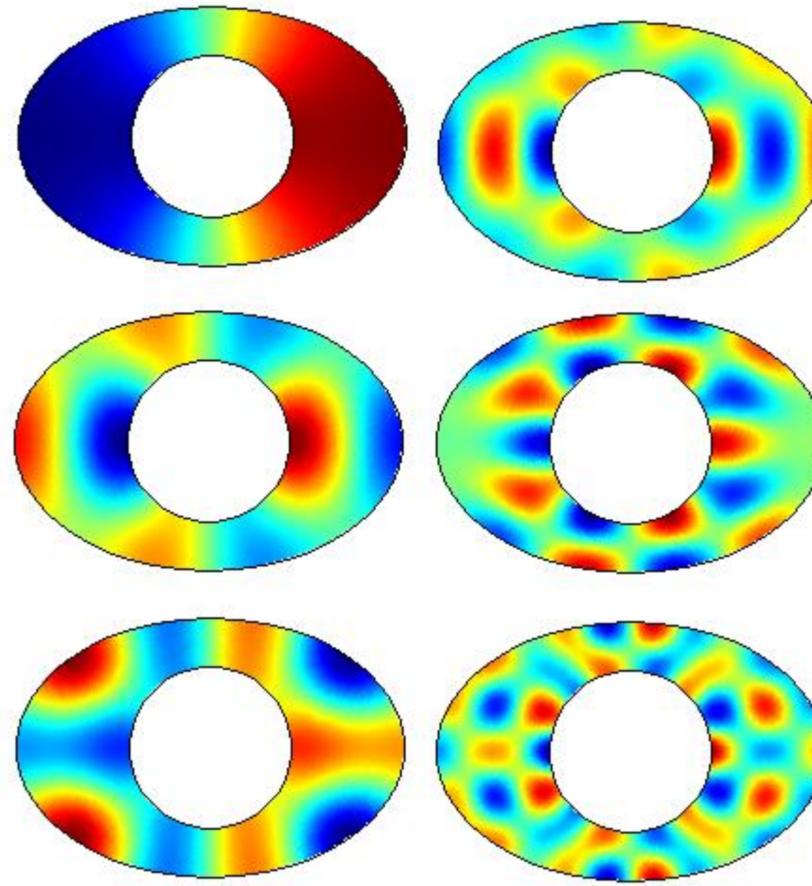


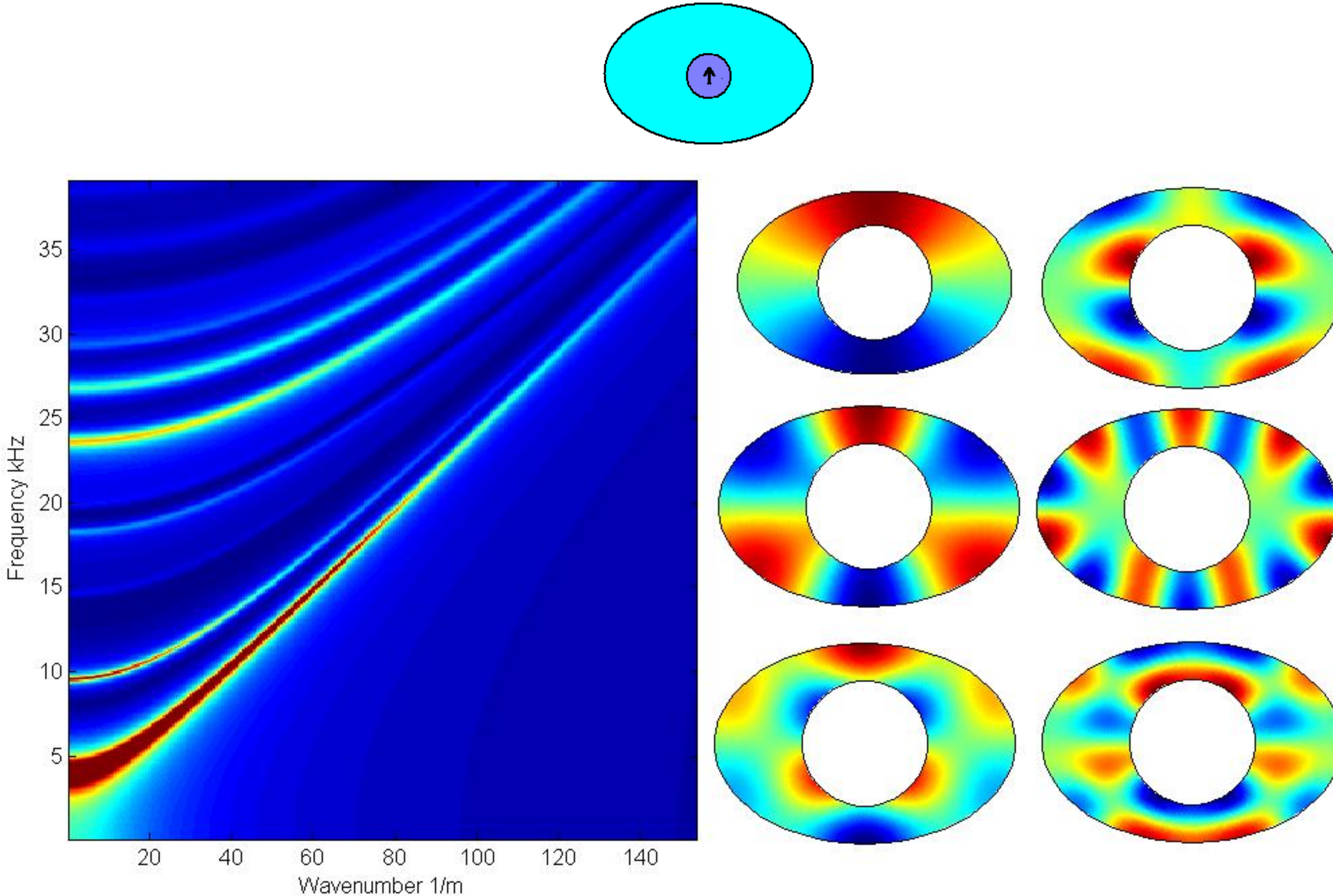
- Section: Ellipse 240X160mm, 2.5 D method
- No analytical solution





Frequency kHz
35
30
25
20
15
10
5
20
40
60
80
100
120
140
Wavenumber 1/m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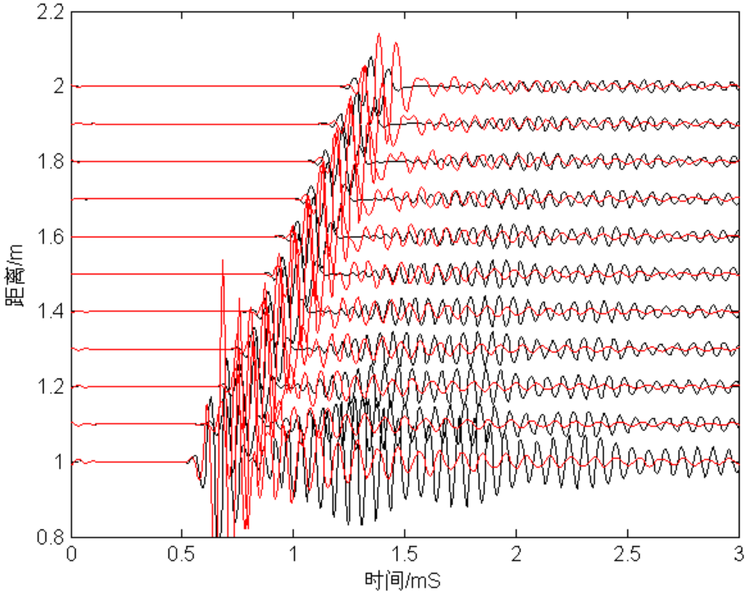
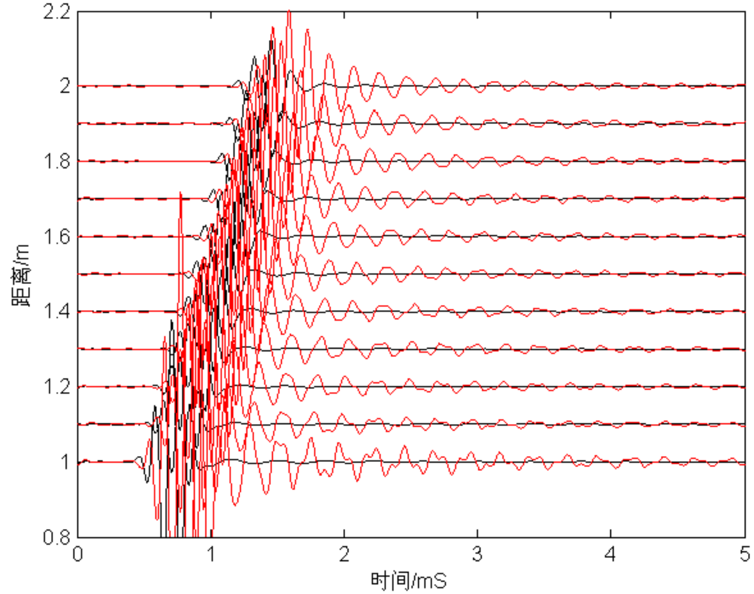
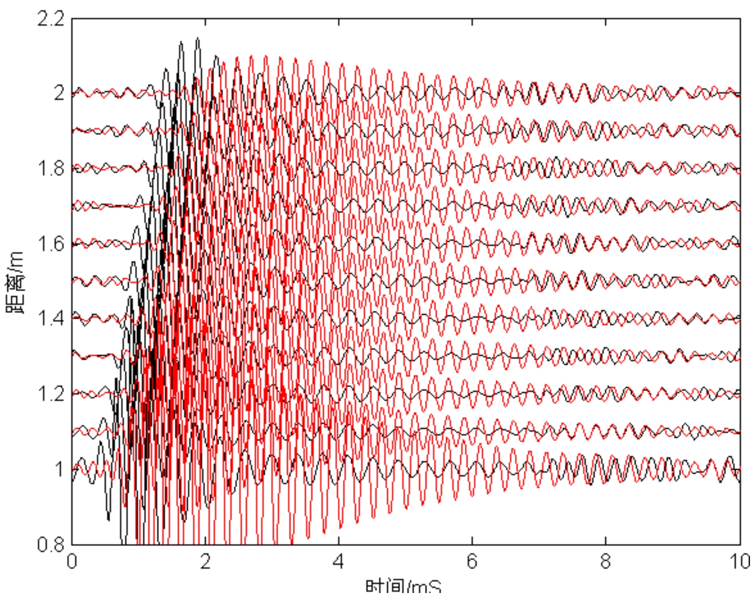
## 14KHz



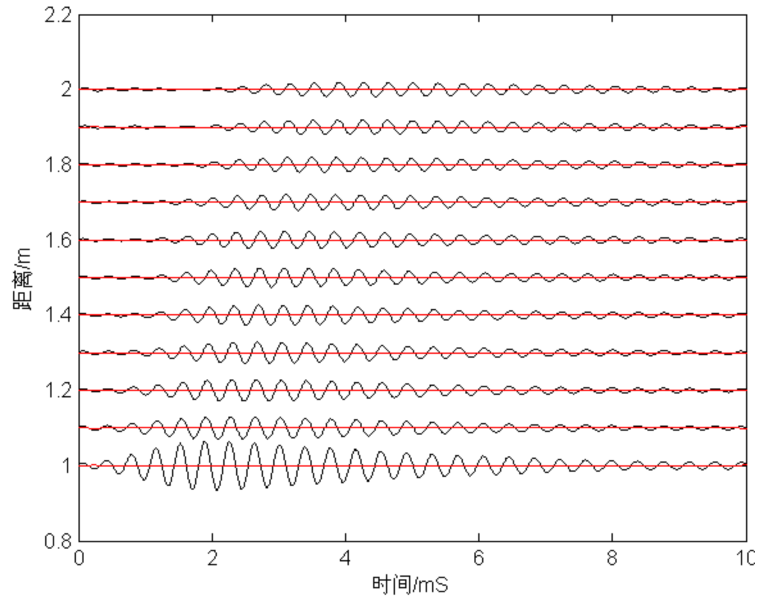

## 8KHz

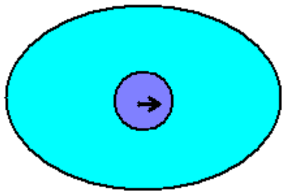



## 4KHz

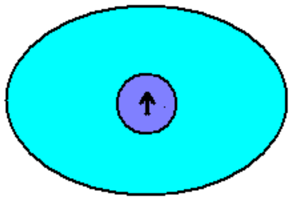



## 2KHz



**Black line**



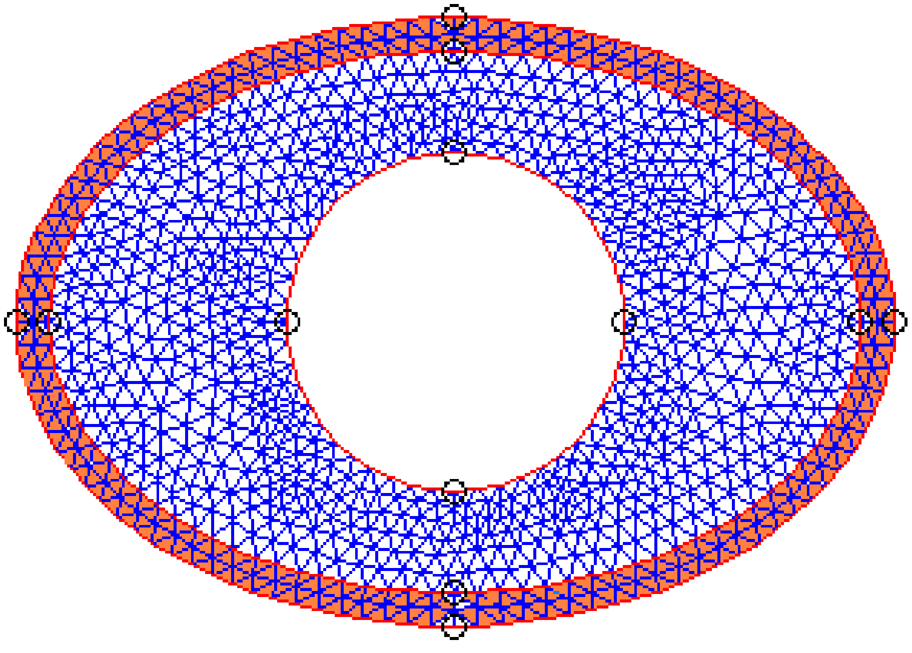
**Red line**

# Conclusion

- The acoustic field in noncirclar wells are calculated by using of the PDE mode of Comsol Multiphysics Software.
- The received waveforms of the dipole tools in noncircular pipes are more complicated than that in the circular pipes because more modes are excited.
- It seems that the pipes of different sections may be used to calibrate logging tools
- However, further works are needed.

# Thank you!

# Steel Wall Pipe



# Solid Medium

- 2.5D motion equations

$$(\lambda+2\mu)U_{1,11}+\lambda U_{2,21}+\lambda ikU_{3,1}+\mu\left(U_{1,22}+U_{2,12}\right)+\mu\left(ikU_{3,1}-k^2U_1\right)+\rho\omega^2U_1=0$$

$$\mu\left(U_{1,21}+U_{2,11}\right)+(\lambda+2\mu)U_{2,22}+\lambda U_{1,12}+\lambda ikU_{3,2}+\mu\left(ikU_{3,2}-k^2U_2\right)+\rho\omega^2U_2=0$$

$$\mu\left(ikU_{1,1}+U_{3,11}\right)+\mu\left(U_{3,22}+ikU_{2,2}\right)+\lambda ik\left(U_{1,1}+U_{2,2}\right)-k^2(\lambda+2\mu)U_3+\rho\omega^2U_3=0$$

$$\boldsymbol{c} = \begin{pmatrix} \begin{pmatrix} \lambda+2\mu & 0 \\ 0 & \mu \end{pmatrix} & \begin{pmatrix} 0 & \lambda \\ \mu & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & \mu \\ \lambda & 0 \end{pmatrix} & \begin{pmatrix} \mu & 0 \\ 0 & \lambda+2\mu \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} \mu & 0 \\ 0 & \mu \end{pmatrix} \end{pmatrix}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{pmatrix} ik\lambda \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ ik\lambda \end{pmatrix} \\ \begin{pmatrix} ik\mu \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ ik\mu \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} \end{pmatrix} \quad \boldsymbol{\beta} = \begin{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{pmatrix} -ik\mu \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ -ik\mu \end{pmatrix} \\ \begin{pmatrix} -ik\lambda \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ -ik\lambda \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} \end{pmatrix}$$

$$\boldsymbol{a} = \begin{pmatrix} \mu k^2 - \rho\omega^2 & 0 & 0 \\ 0 & \mu k^2 - \rho\omega^2 & 0 \\ 0 & 0 & (\lambda + 2\mu)k^2 - \rho\omega^2 \end{pmatrix} \qquad \boldsymbol{\gamma} = 0$$