COMSOL CONFERENCE
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A Black-Oil Model for Primary and Secondary Oil-Recovery in Stratified Petroleum Reservoirs

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Objective

❖ Development of the **black-oil model** in COMSOL Multiphysics platform.

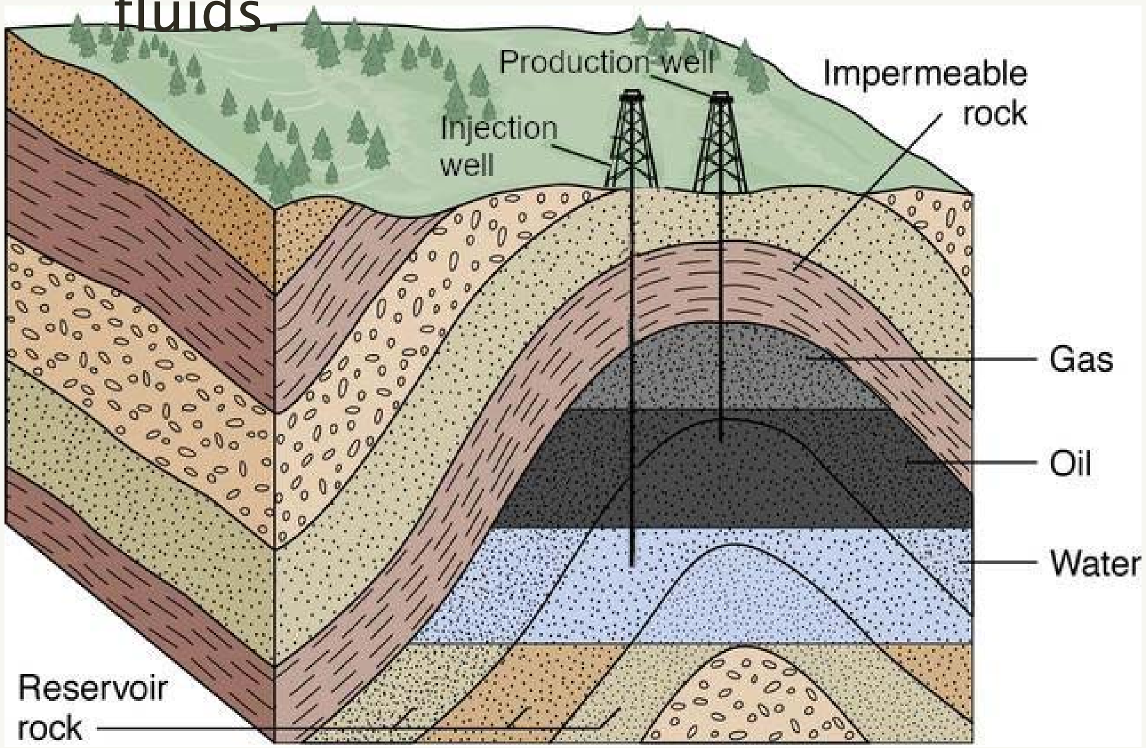
❖ Implementation of a **numerically stable formulation.**

❖ Oil-recovery **scenarios**:

- Pressure depletion with solution-gas drive mechanism
- Waterflooding
- Gas injection

The Physical Problem

➢ **Recovery** of **financially significant fluids.**



Primary oil-recovery (Pressure depletion):

- Natural reservoir energy
- **Solution-gas drive** mechanism : Expansion of oil & evolved solution gas

Secondary oil-recovery:

- Pressure maintenance
- Injection fluids: **water, gas**
- Oil displacement

Enhanced oil-recovery (EOR):

- Thermal processes, miscible fluids, surfactants

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The "Black-Oil Model" Approach

- Prediction of **oil production** dynamics & reservoir conditions.

Reservoir Conditions
$p = 270\ bar, T = 75^0C$

$$B_\beta = \frac{V_\beta^{RC}}{V_\beta^{SC}},$$
$$\beta = o, g, w$$

Surface Conditions
$p = 1\ bar, T = 15^0C$

Typically:
$B_o > 1$
$B_g \ll 1$
$B_w \approx 1$

Oil phase → Oil phase

Oil phase → evaporation → Gaseous phase

Gaseous phase → Gaseous phase

Water phase → Water phase

$$R_{so} = \frac{V_{dg}^{SC}}{V_o^{SC}}$$

The "Black-Oil Model" System

Balance equations on "standard volumes":

Gas phase ("g") :

$$\varphi \frac{\partial}{\partial t}\left(\frac{S_g}{B_g} + \frac{R_{so} S_o}{B_o}\right) + \nabla \cdot \left(\frac{1}{B_g}\boldsymbol{u}_g + \frac{R_{so}}{B_o}\boldsymbol{u}_o\right) = q_g$$

Oil phase ("o") :

$$\varphi \frac{\partial}{\partial t}\left(\frac{S_o}{B_o}\right) + \nabla \cdot \left(\frac{1}{B_o}\boldsymbol{u}_o\right) = q_o$$

Water phase ("w") :

$$\varphi \frac{\partial}{\partial t}\left(\frac{S_w}{B_w}\right) + \nabla \cdot \left(\frac{1}{B_w}\boldsymbol{u}_w\right) = q_w$$

$k_{r\beta}(S_\beta)$

Darcy's law :

$$\boldsymbol{u}_\beta = -\boldsymbol{K}\frac{k_{r\beta}}{\mu_\beta}(\nabla p_\beta - \rho_\beta \boldsymbol{g}), \quad for \quad \beta = \boldsymbol{g}, \boldsymbol{o}, \boldsymbol{w}$$

$\mu_\beta(p_\beta)$

Saturation constraint:

$$S_g + S_o + S_w = 1$$

S:	Saturation
$\boldsymbol{u}$:	Velocity
B:	Formation volume factor
R_{so}:	Solution gas–oil ratio
$\boldsymbol{K}$:	Intrinsic permeability
k_r:	Relative permeability
μ:	Viscosity
ρ:	Density

$$\rho_g = \frac{\rho_{Gs}}{B_g}$$

$$\rho_o = \frac{\rho_{Os} + R_{so}\rho_{Gs}}{B_o}$$

$$\rho_w = \frac{\rho_{Ws}}{B_w}$$

Numerical Challenge

Finite Element Method:

Non-linearity & coupling weakened

Flow & convection dominate ➡ Very steep interface gradients

Numerical instabilities

⬇

More stable formulation using **terms rearrangement**!

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Phase Formulation (Chen, 2000)

Pressure variable : $p = p_o$ Total velocity : $\boldsymbol{u} = \sum \boldsymbol{u}_\beta$, $for\ \beta = g, o, w$

1 Pressure equation :

$$\nabla \cdot \boldsymbol{u} = \sum B_\beta \left(q_\beta - \varphi S_\beta \frac{\partial}{\partial t}\left(\frac{1}{B_\beta}\right) - \boldsymbol{u}_\beta \cdot \nabla\left(\frac{1}{B_\beta}\right)\right) - B_g \left(R_{so} q_o + \frac{\varphi S_o}{B_o}\frac{\partial R_{so}}{\partial t} + \frac{1}{B_o}\boldsymbol{u}_o \cdot \nabla R_{so} \right)$$

General Form PDE

2 Saturation equations :

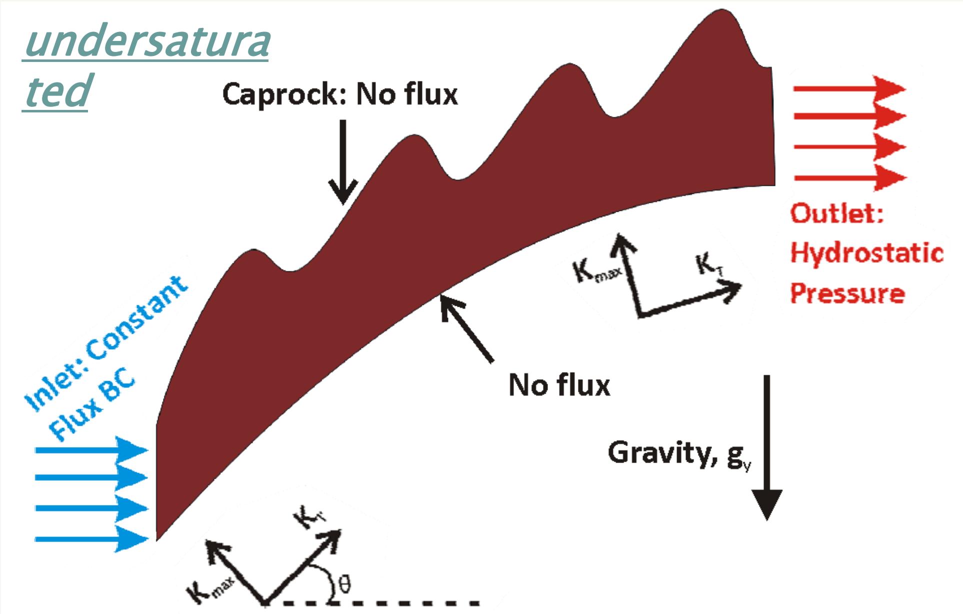
$$\varphi \frac{\partial S_\alpha}{\partial t} + \nabla \cdot \boldsymbol{u}_\alpha = B_\alpha \left(q_\alpha - \varphi\, S_\alpha \frac{\partial}{\partial t}\left(\frac{1}{B_\alpha}\right) - \boldsymbol{u}_\alpha \cdot \nabla\left(\frac{1}{B_\alpha}\right)\right)$$

Coefficient Form PDE

$$\boldsymbol{u}_\alpha = f_\alpha \boldsymbol{u} - \boldsymbol{K} f_\alpha \sum \lambda_\beta \left((\rho_\beta - \rho_\alpha)\boldsymbol{g} \right), \qquad for\ \alpha = o, w$$

where $G_\lambda = \boldsymbol{g} \sum f_\beta \rho_\beta$, $f_\beta = \frac{\lambda_\beta}{\lambda}, \lambda_\beta = \frac{k_{r\beta}}{\mu_\beta}, \lambda = \sum \lambda_\beta$

Applications on Typical Reservoir Structures



Initial Conditions

$p(t_0) = p_{in} = 10^5 + \rho_w \cdot g_y(2860 - y)[Pa]$
$S_o(t_0) = 1 - S_{wc}$
$S_w(t_0) = S_{wc} = 0.1$

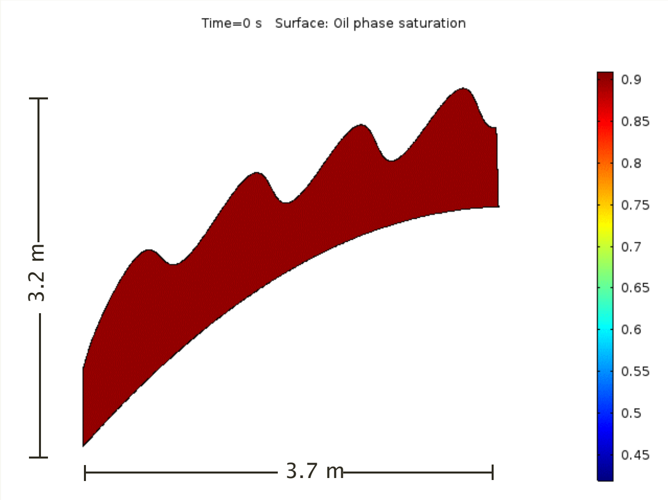
negligible capillary forces

$$\boldsymbol{u_\beta} = (u_{\beta x}, u_{\beta y}),$$
$$u_{\beta x} = \lambda_\beta \left(k_{11}\frac{\partial p}{\partial x} + k_{12}\left(\frac{\partial p}{\partial y} + \rho_\beta g_y\right)\right)$$
$$u_{\beta y} = \lambda_\beta \left(k_{21}\frac{\partial p}{\partial x} + k_{22}\left(\frac{\partial p}{\partial y} + \rho_\beta g_y\right)\right),$$
$$for\ \beta = g, o, w$$

$$\boldsymbol{K} = \begin{bmatrix} k_{max}\cos^2\theta + k_T sin^2\theta & k_{max} cos\theta sin\theta - k_T cos\theta sin\theta \\ k_{max} sin\theta cos\theta - k_T sin\theta cos\theta & k_{max}\sin^2\theta + k_T\cos^2\theta \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

Primary Oil-Recovery: Solution-gas Drive Mechanism

Evolution of Oil Phase Saturation



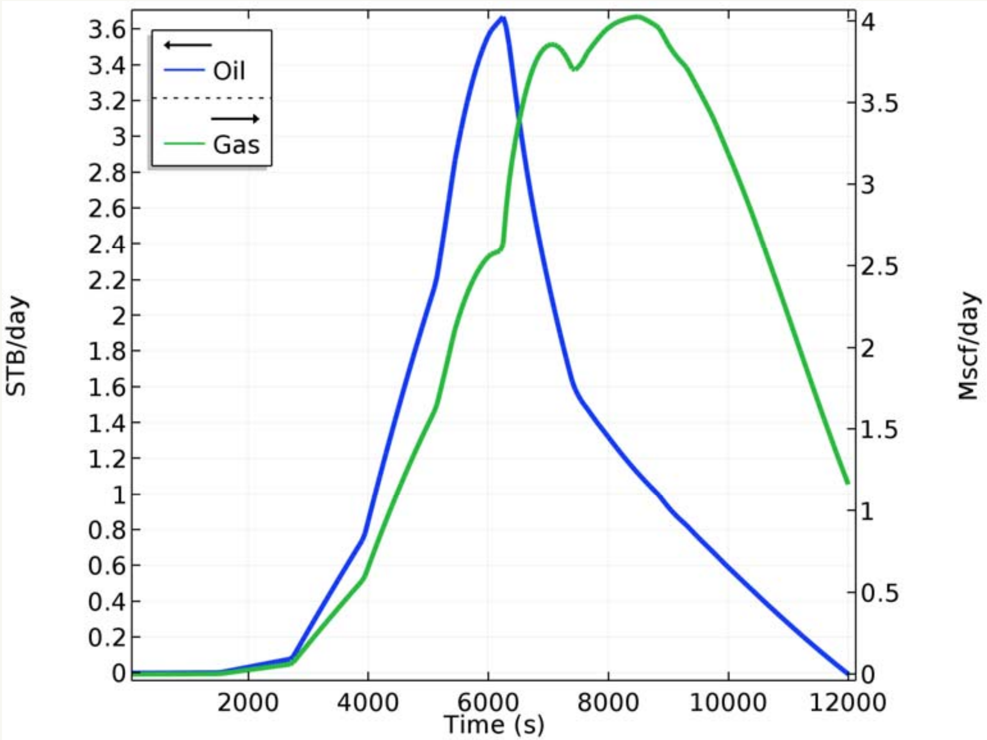
Boundary conditions

Outlet

$$p = p_{out} = p_{in} - 0.2 \cdot p_{in} \cdot flc2hs(t - t_1, t_2)$$
$$-\boldsymbol{n} \cdot (-\varepsilon \nabla S_o + \boldsymbol{u}_o) = -\boldsymbol{u}_o$$
$$-\boldsymbol{n} \cdot (-\varepsilon \nabla S_w + \boldsymbol{u}_w) = -\boldsymbol{u}_w$$

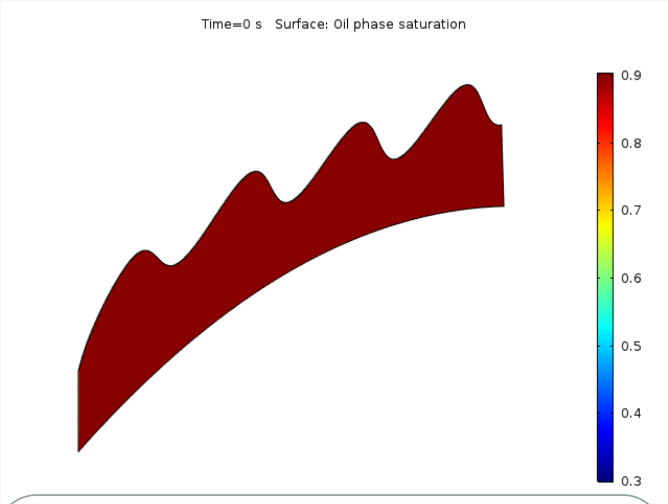
where $\varepsilon = 10^{-7} m^2/s$
the numerical diffusion

Oil & Gas Production Rates



Secondary Oil-Recovery: Waterflooding

Evolution of Oil Phase Saturation

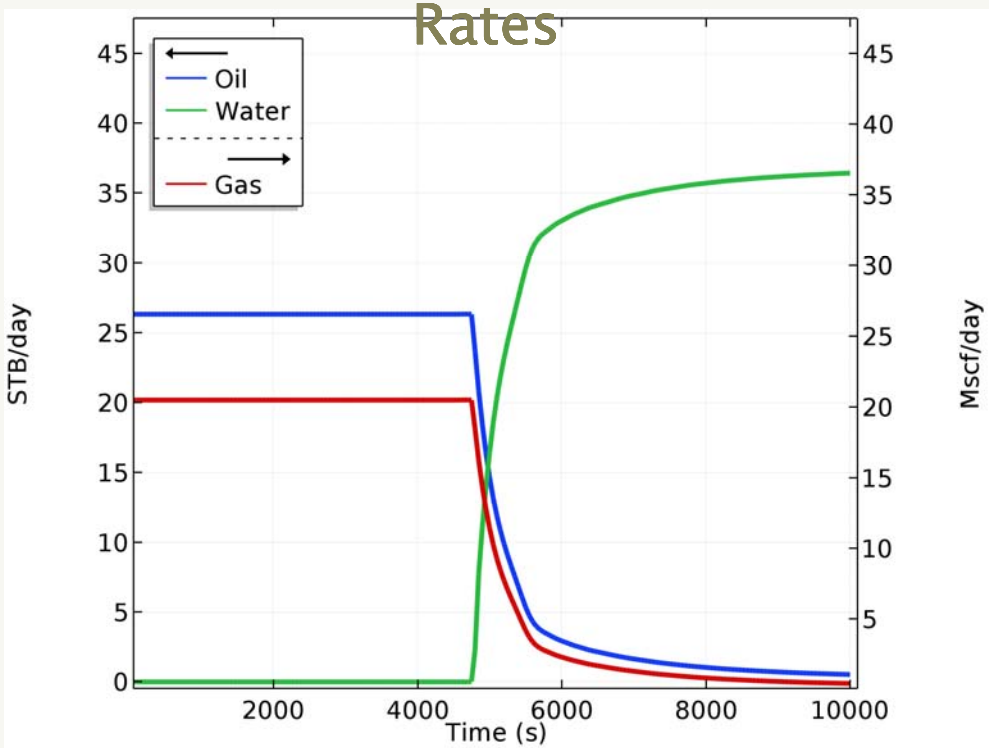


Boundary conditions

Inlet	Outlet
$-\boldsymbol{n}\cdot\boldsymbol{u} = u_{in}$	$p = p_{out} = p_{in}$
$S_o = S_{or} = 0.3$	$-\boldsymbol{n}\cdot(-\varepsilon\nabla S_o + \boldsymbol{u_o}) = -\boldsymbol{u_o}$
$S_w = 1 - S_{or}$	$-\boldsymbol{n}\cdot(-\varepsilon\nabla S_w + \boldsymbol{u_w}) = -\boldsymbol{u_w}$

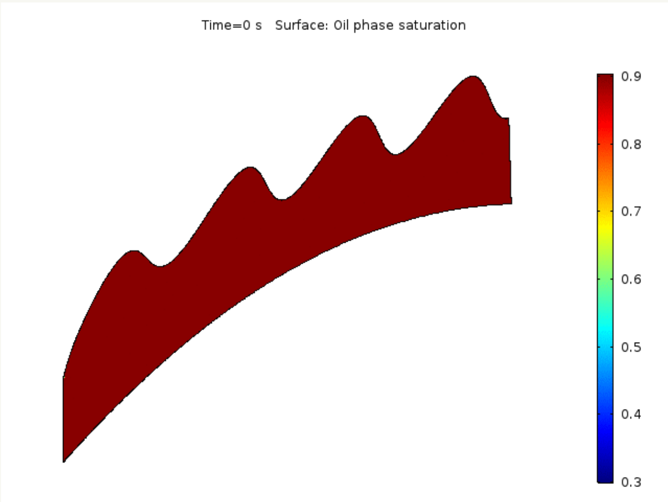
where $u_{in} = 10^{-4}\frac{m}{s}$,
$\varepsilon = 5\cdot 10^{-6} m^2/s$

Oil, Gas & Water Production Rates



Secondary Oil-Recovery: Gas Injection

Evolution of Oil Phase Saturation

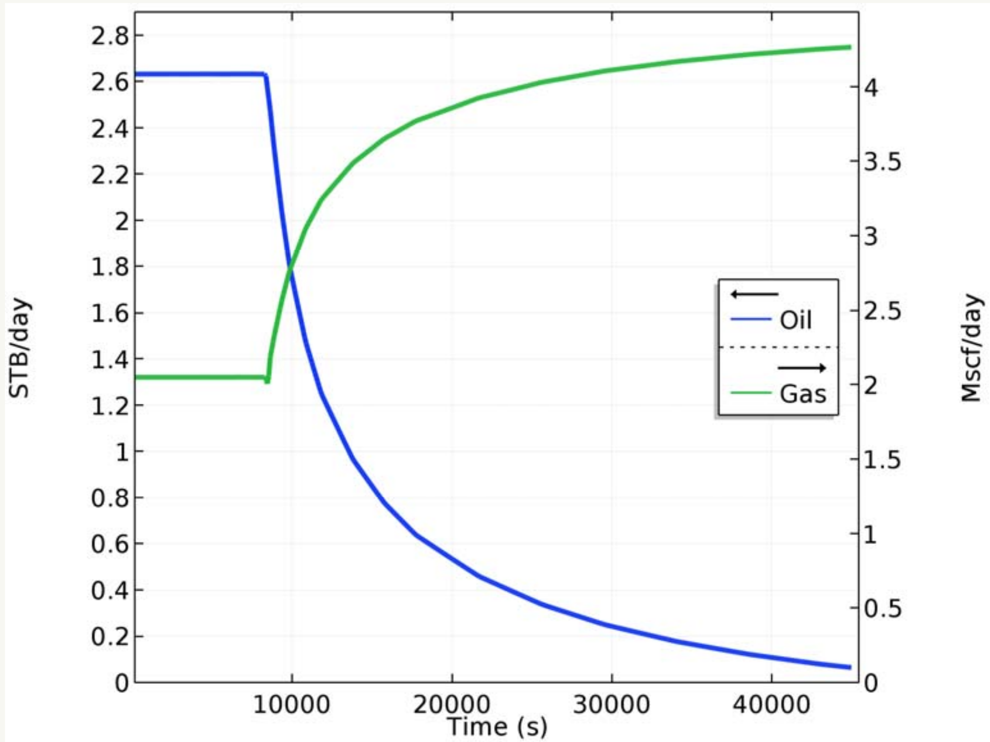


Boundary conditions

Inlet	Outlet
$-n \cdot u = u_{in}$	$p = p_{out} = p_{in}$
$S_o = S_{or}$	$-n \cdot (-\varepsilon \nabla S_o + u_o) = -u_o$
$S_w = S_{wc}$	$S_w = S_{wc}$

where $u_{in} = 10^{-5} \frac{m}{s}$,
$\varepsilon = 8 \cdot 10^{-7} m^2/s$

Oil & Gas Production Rates



Conclusions

- ✓ Successful representation of the physical phenomenon.
- ✓ Buoyancy and gravity effects visually verified.
- ✓ Reliable tool for the estimation of fluids recovery.
- ➢ Simulation of more complex problems in EOR processes by implementing also the appropriate PDEs.

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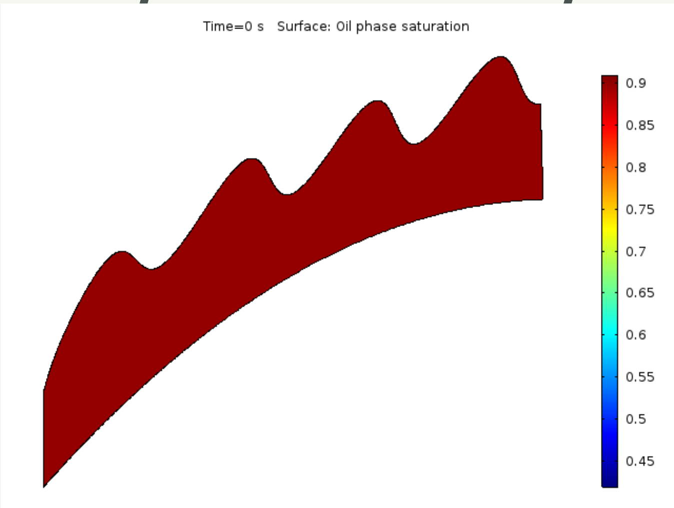
THANK YOU!

Primary Oil-Recovery: Solution-gas Drive Mechanism

a)



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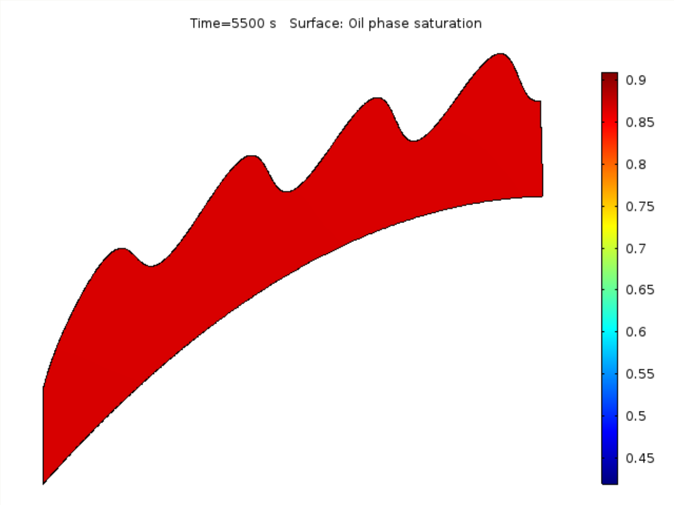

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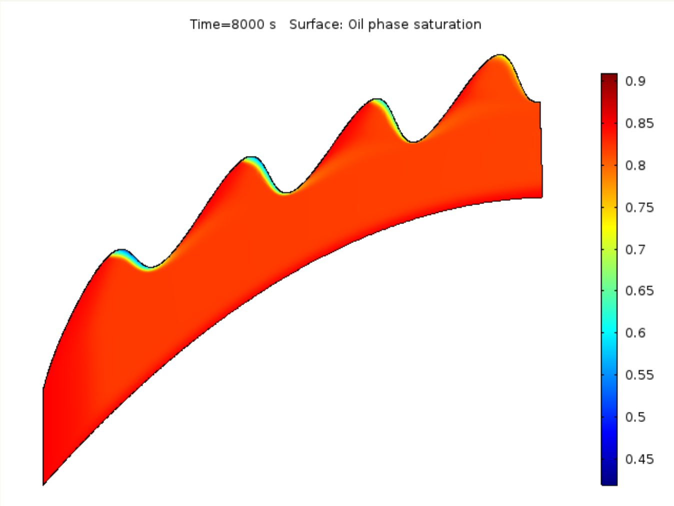

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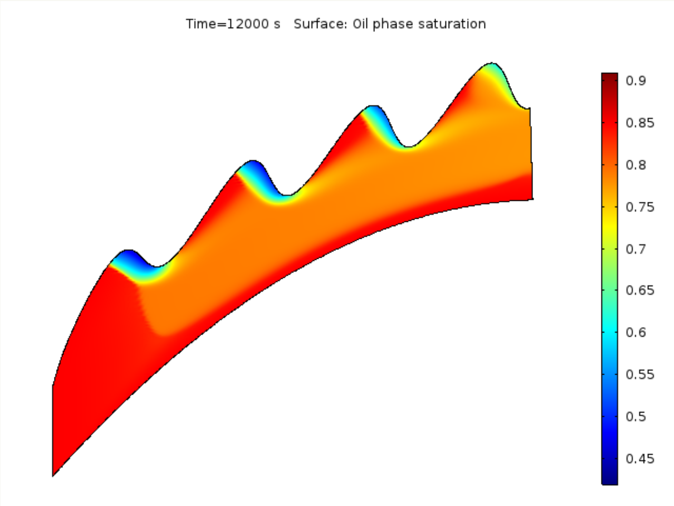
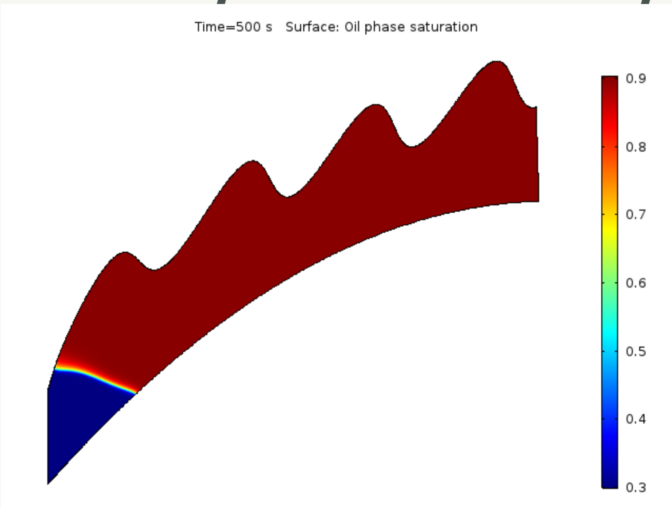


Secondary Oil-Recovery: Waterflooding

a)



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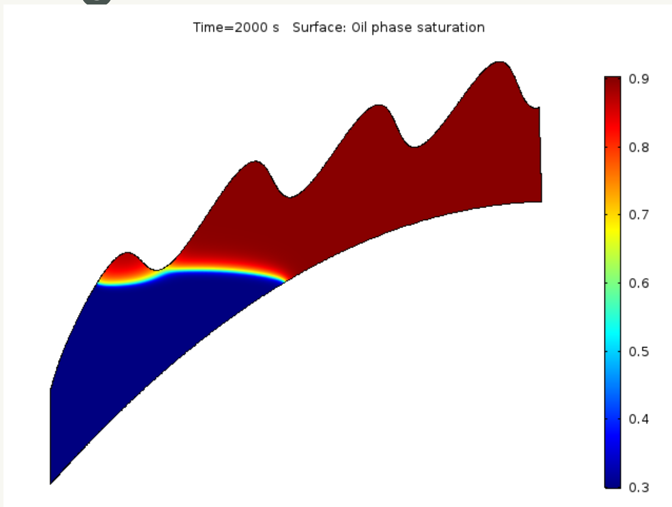

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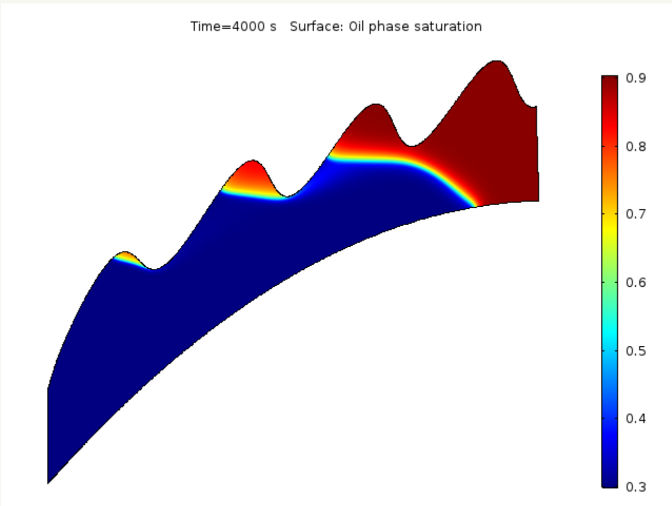
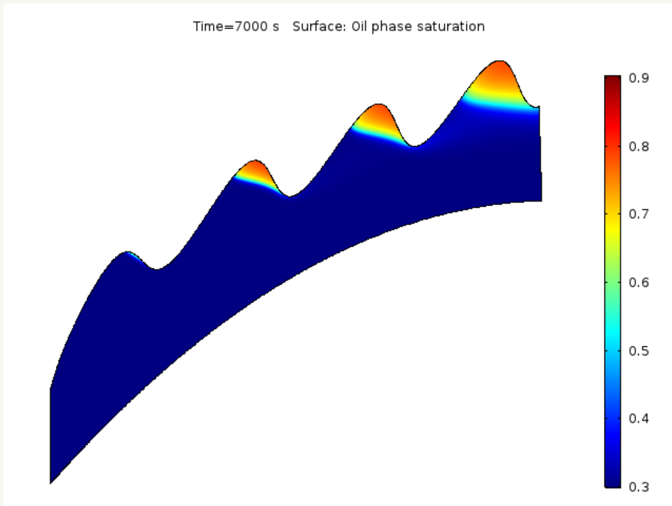


d)



Secondary Oil-Recovery: Gas Injection

a) b) c) d)