COMSOL
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Multiphysics Approach to Sediment Transport in Shallow Water

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Flash Floods

www.pinterest.com

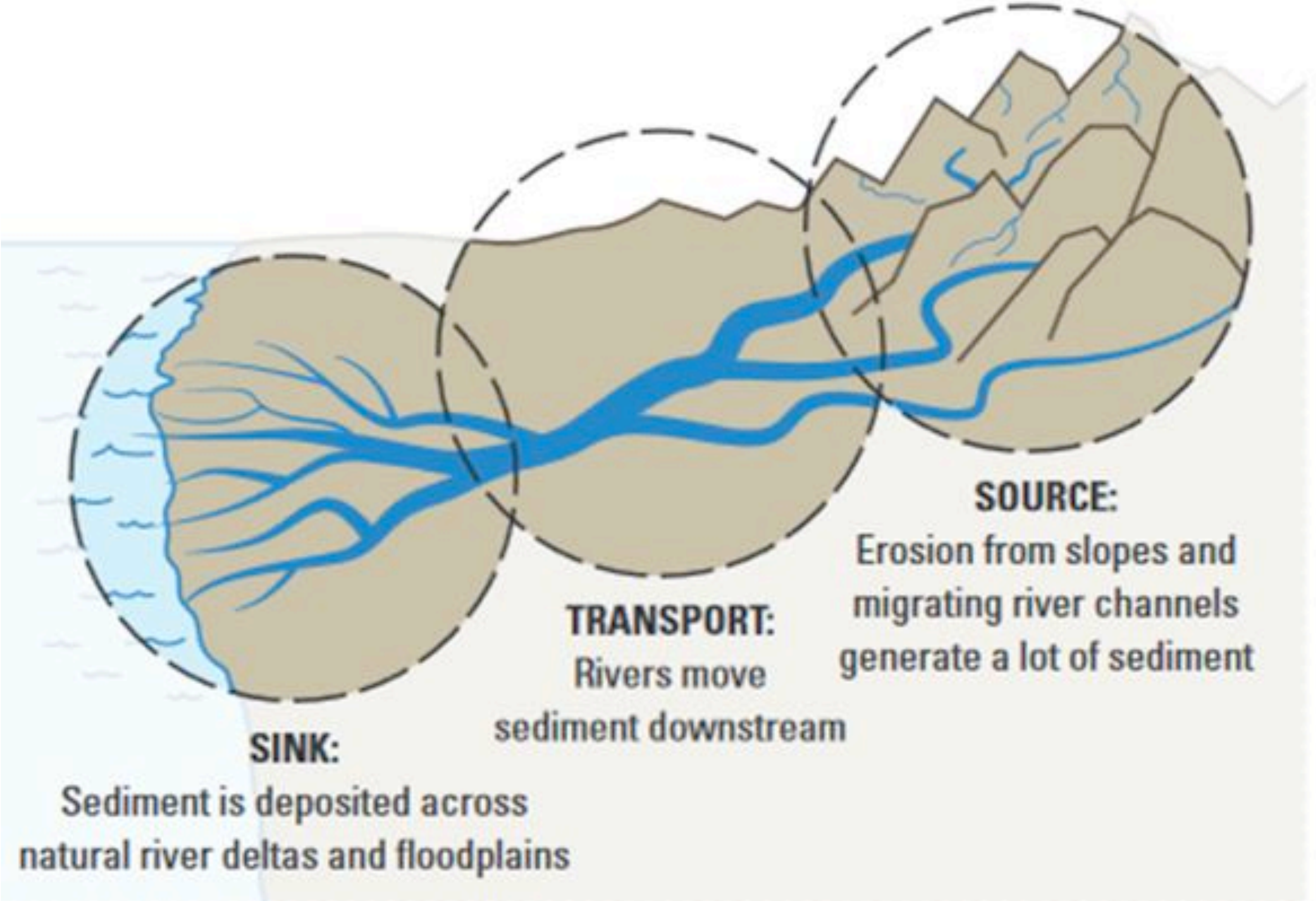
Rapid flooding due to

- heavy rain in a watershed
- meltwater of snow and ice
- failure of a protection structure

Time scale: few hours maximum

Sediment Transport

Sediment Migration



SOURCE:
Erosion from slopes and migrating river channels generate a lot of sediment

TRANSPORT:
Rivers move sediment downstream

SINK:
Sediment is deposited across natural river deltas and floodplains

Sediment is the sand, mud, and pebbles that were once solid rock.

Sediment flows in tributary streams and river channels of the Skagit, from the Cascade Mountains to Skagit Bay and Puget Sound.

http://www.skagitclimatescience.org/skagit-impacts/sediment/

Sediment Transport

www.fondriest.com/environmental-measurements/parameters/hydrology/sediment-transport-deposition/

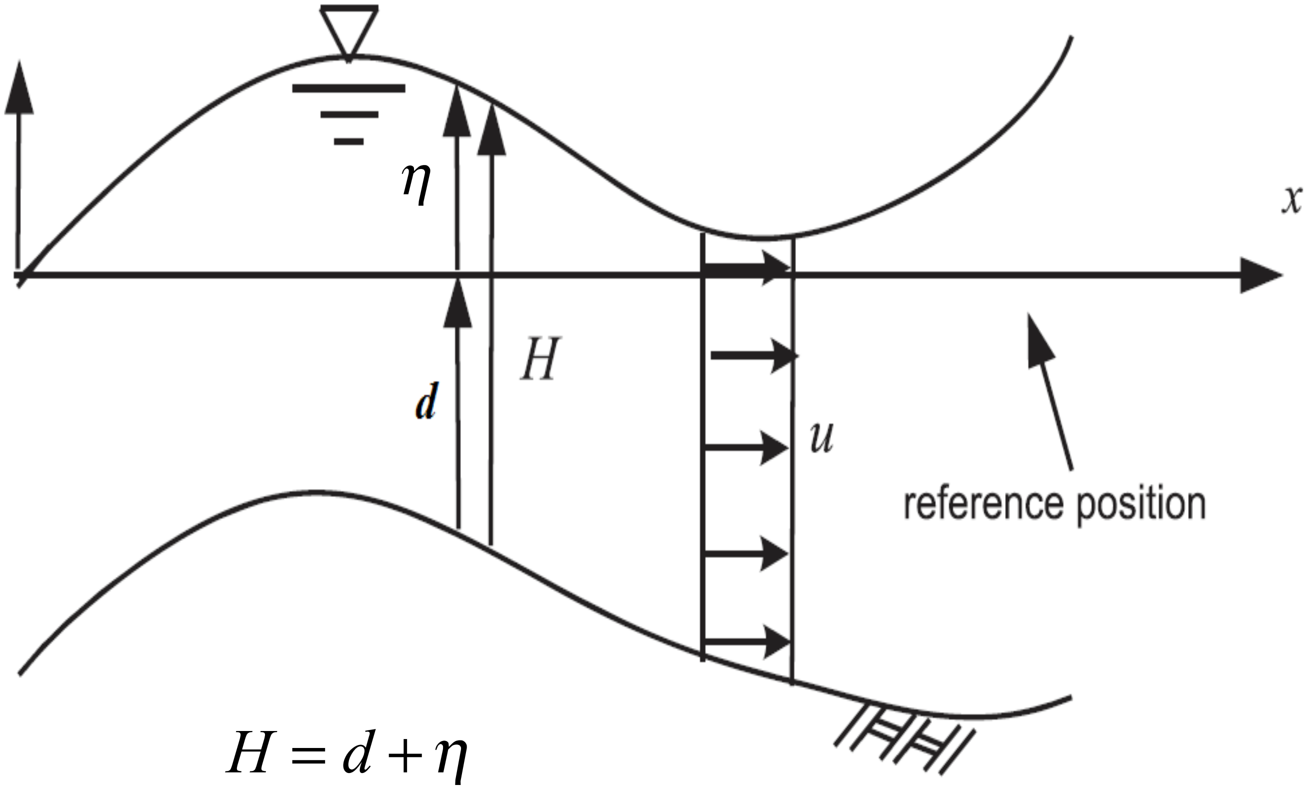
Reduced Dimension

3D phenomena are simplified to 2D (floods) or 1D (streams).

The 1D or 2D description

- is derived from the 3D equations for mass and momentum conservation
- Assumes that vertical length scale is much smaller than the horizontal scales
- is not valid in case of non-negligible vertical flux components
- is usually used for shallow waters

Depth-Averaged Flow



$$H = d + \eta$$

Shallow Water Equations

SWE

Saint-Venant Equations

- Volume Conservation

$$\frac{\partial \eta}{\partial t} + \nabla \cdot \left(H\mathbf{u} \right) = 0$$

- Momentum Conservation

$$\frac{\partial \mathbf{u}}{\partial t} + \left(\mathbf{u} \cdot \nabla \right) \mathbf{u} + g \nabla H - \mathbf{F} = 0$$

with with total water depth H, water height above reference height η, velocity vector $\mathbf{u}$, acceleration and due to gravity g and vector of outer forces $\mathbf{F}$

Extension

Hydraulics with friction on the walls

$$\frac{\partial \mathbf{u}}{\partial t} + \left(\mathbf{u}\cdot\nabla\right)\mathbf{u} + g\nabla H + g\eta n^2 \frac{\left|\mathbf{u}\right|}{\eta^{4/3}}\mathbf{u} - \mathbf{F} = 0$$

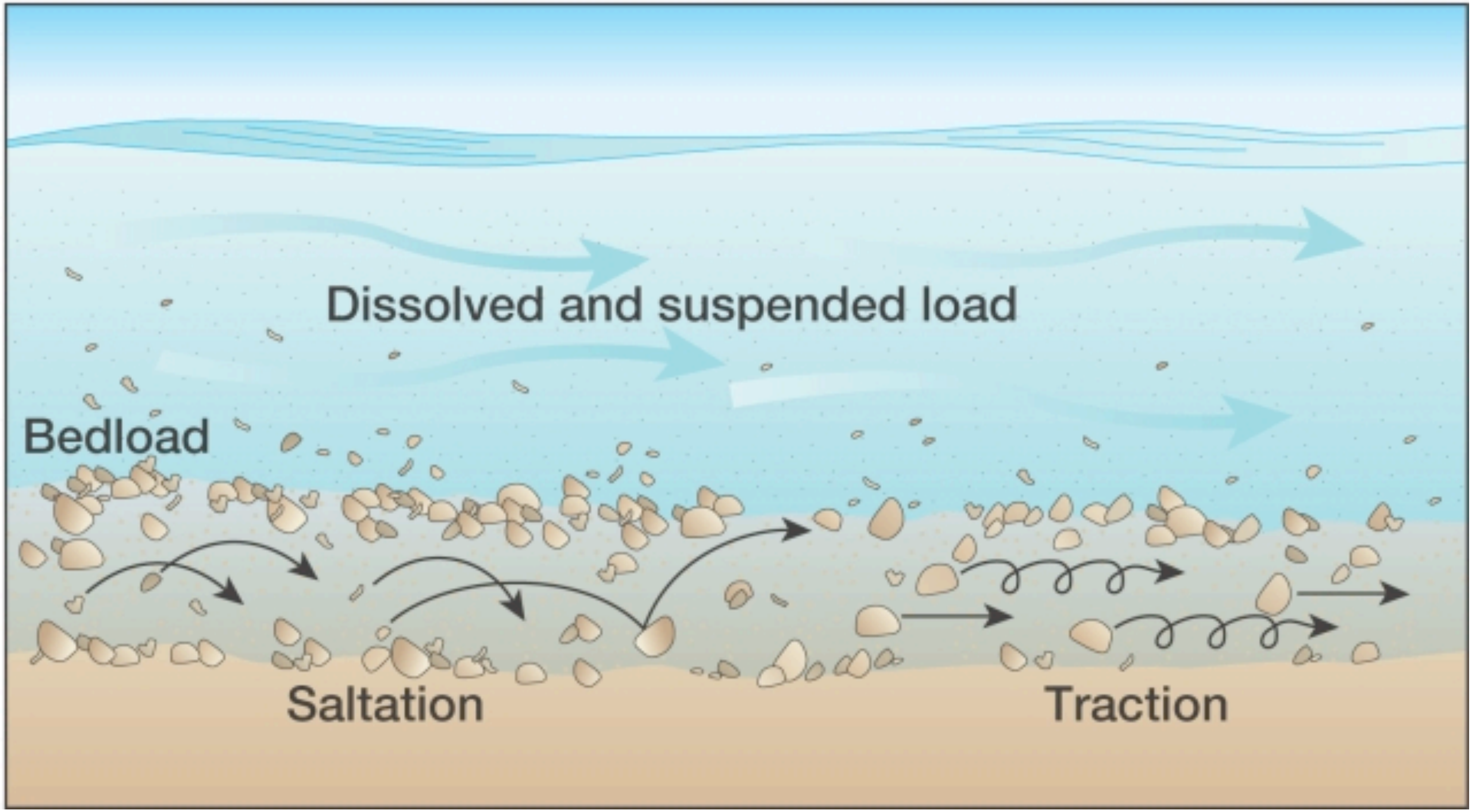
with Manning coefficient n

(Brufau & García-Navarro 2000, Duran 2015)

Implemented in COMSOL Multiphysics as physics mode by Schlegel (2012)

Solute Transport in Streams

Mechanisms of sediment transport (from McKnight and Hess, 2000)



Suspended (Particulate, Dissolved) Load

Transport equation (1D, 2D)

$$\frac{\partial c}{\partial t} - \nabla\left(\mathbf{D}\nabla c\right) + \left(\mathbf{u}\cdot\nabla\right)c - \frac{1}{H}\left(E - D\right) = 0$$

with dispersion tensor $\mathbf{D}$, velocity $\mathbf{u}$, source term E and sink term D

(Cao *et al*. 2004, Li & Duffy 2011, Rowan & Seaid 2017)*

In cases several different classes have to be taken into account

*except for the dispersion term

Bedload

Transport equation (1D, 2D)

$$\frac{\partial d}{\partial t} - \frac{1}{1-\theta}\left(E - D\right) = 0$$

with depth d below reference and porosity θ .

E describes sedimentation of supended load, D the re-mobilization of bedload

(Cao *et al*. 2004, Li & Duffy 2011)

Closure 1

Sedimentation (Empirical formula)

$$D = \beta v_s c_s \left(1 - c_s\right)^m$$

with sedimentation parameter β , settling velocity v_s and exponent m. For particular load the settling velocity depends on grain size!

$m = 0$ (Cao *et al*. 2004, Li & Duffy 2011)

$m = 1.4$ (Rowan & Seaid 2017)

Closure 2

Re-suspension (Empirical formula)

$$E = \alpha \cdot \mathrm{sign}\left(\Theta - \Theta_c\right) H \left|\mathbf{u}\right|$$

$$\Theta = u_*^2 / sg\delta$$

$$u_* = \sqrt{gh\left(S_{fx}^2 + S_{fy}^2\right)}$$

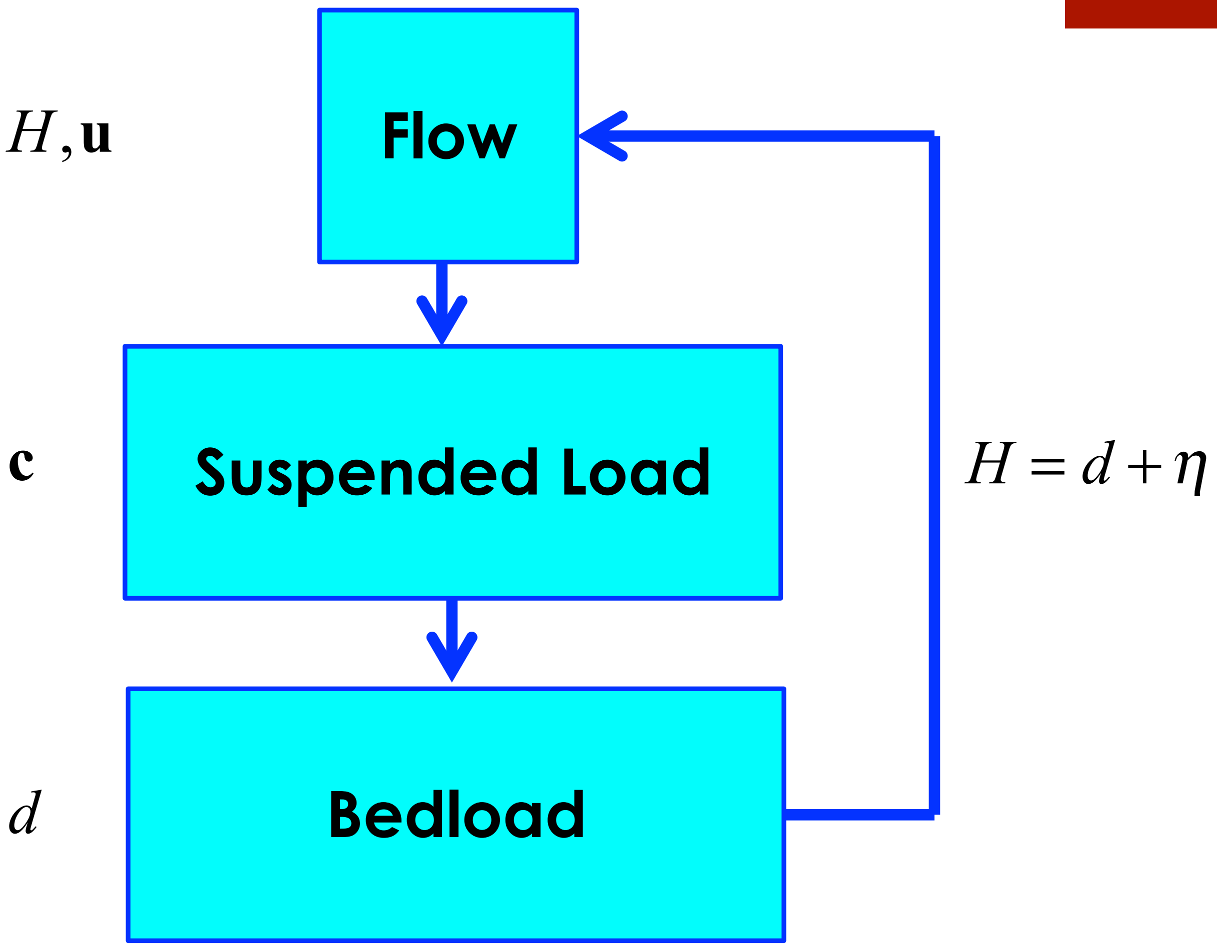
$$S_{fx} = nu_x \left|\mathbf{u}\right| / h^{4/3} \qquad S_{fy} = nu_y \left|\mathbf{u}\right| / h^{4/3}$$

$$s = \rho_s / \rho_f - 1$$

with coefficient α and Shields parameters Θ and Θ_c,

friction velocity u_*, friction slopes S_{fx}, S_{fy}, specific gravity s

Multiphysics Coupling


$H, \mathbf{u}$

Flow

$\mathbf{c}$

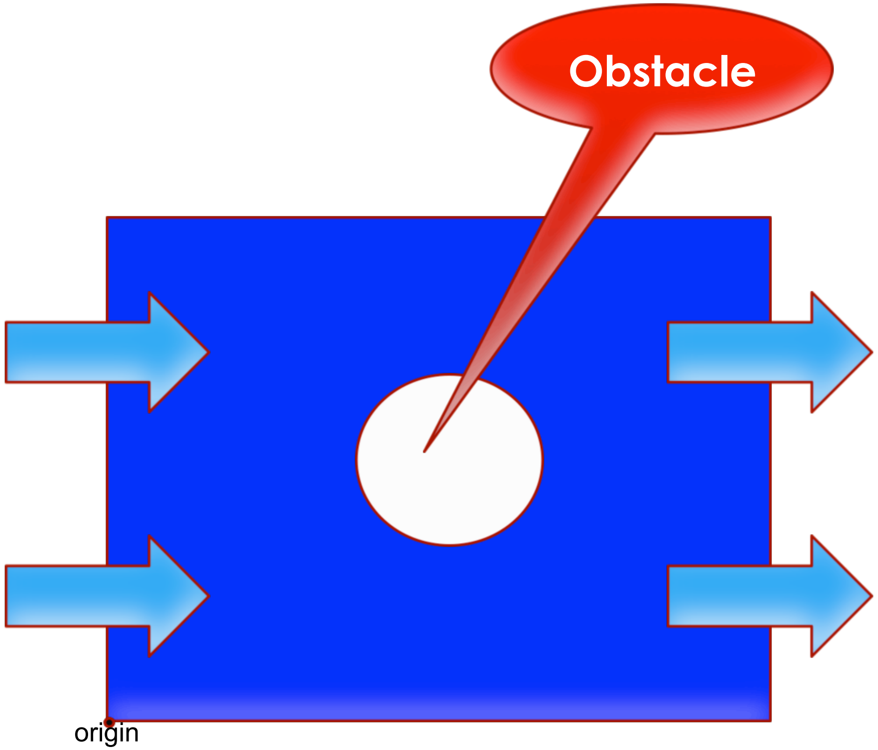
Suspended Load

d

Bedload

$H = d + \eta$

Flow past an Obstacle



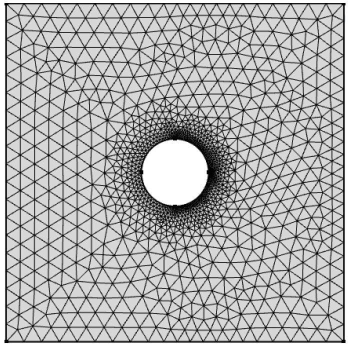
Initial condition:
high water level upstream (left), low water level downstream (right)
at simulation time zero the dam disappears

Parameter Set

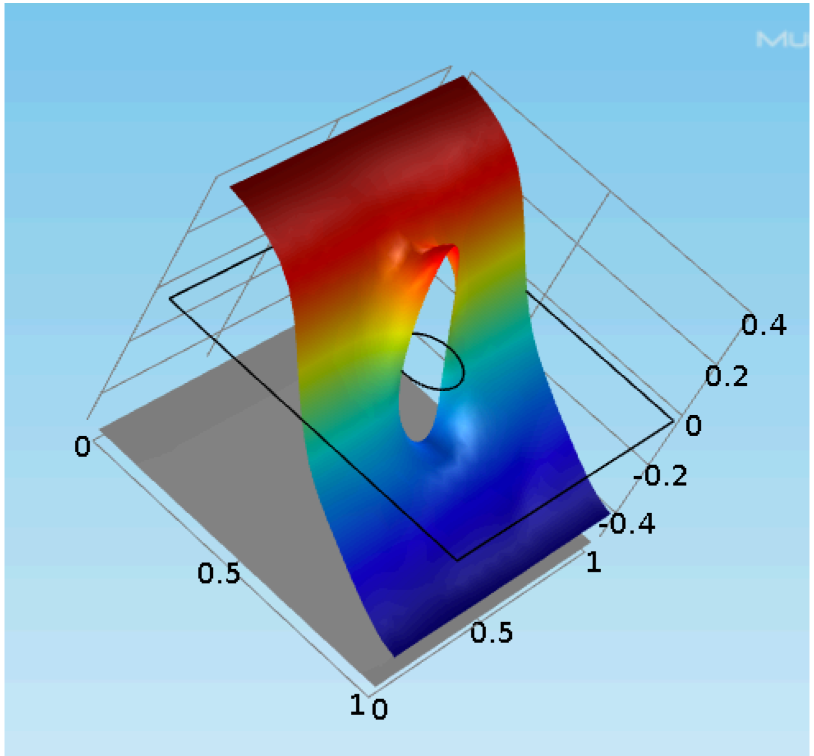
Parameter, Symbol	Value, Unit	Parameter, Symbol	Value, Unit
Length	1 m	Initial particulate load	0
Width	1 m	Settling velocity v_s	0.01 m/s
Obstacle radius	0.1 m	Particle diffusivity	10^{-9} m^2/s
Initial bed below reference	0.5 m	Turbulent viscosity υ	0.0025 m^2/s
Initial water table above reference	0.5 m	Turbulent Schmidt number Sc	0.71
Inflow water table above reference	1 m	Critical Shields parameter Θ_c	0.4
Velocity at outlet	1 m/s	Particle diameter δ	0.0001 m
Manning parameter n	0.03 s/m$^{1/3}$	Re-suspension parameter α	5 10^{-4}
Froude number	0.26	Specific gravity ρ_s/ρ_f	2.65

COMSOL Model

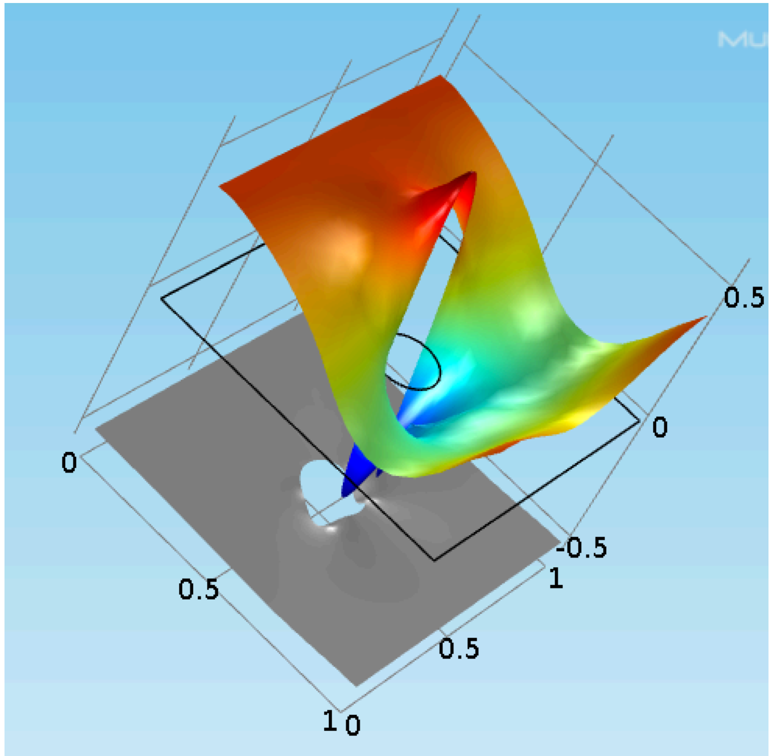
- Model set-up: There is a no-flow no-slip condition along walls. The Manning friction coefficient is *n* = 0.01.
- Elements: 2. order
- Stabilization: consistent
- The Finite Element mesh is refined at the obstacle boundaries. Constructing the mesh a maximum element side length of only 0.01 m at the upstream side, and of 0.005 m at the downstream side was allowed. The resulting mesh, consists of 2476 elements. The discretization of the entire system of coupled differential equations has 10468 degrees of freedom



Results 1

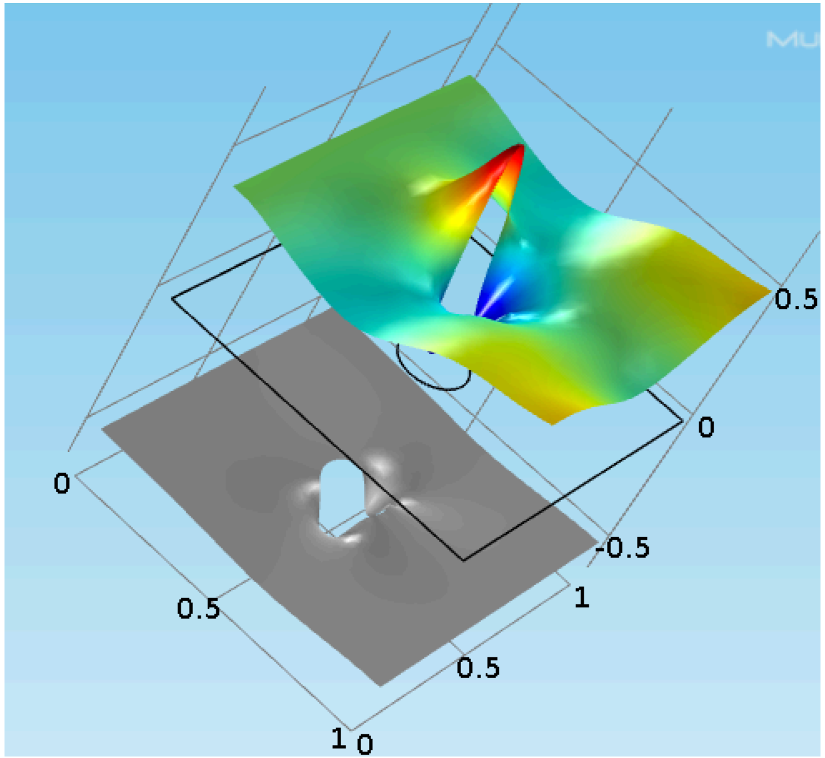


$t = 0.1$ s

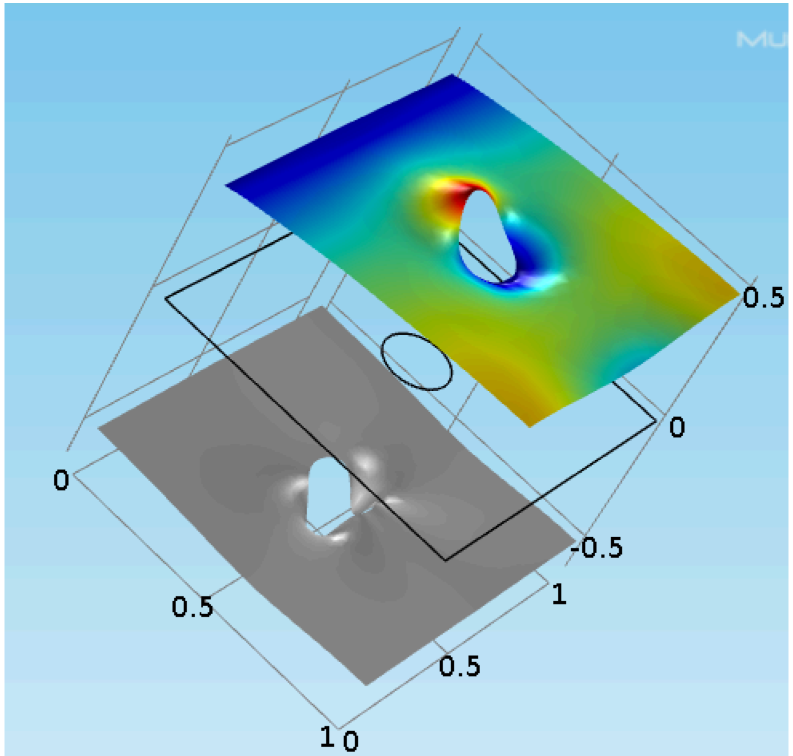


$t = 0.3$ s

Results 2

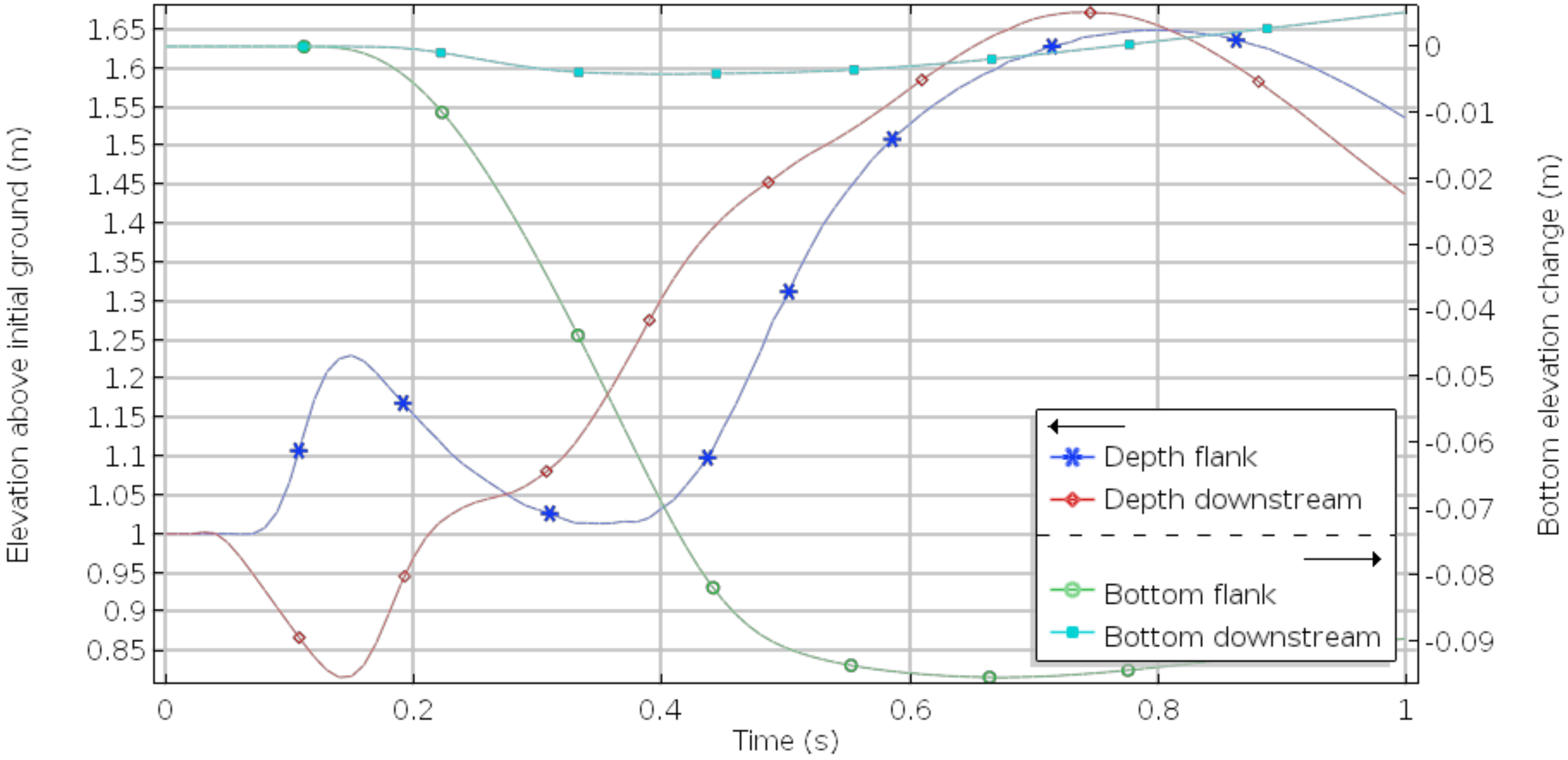


t = 0.5 s



t = 0.9 s

Results 3



Water table and bottom changes at selected locations

Comparison with reality

Depression and sedimentation around obstacle, view upstream in Wadi Abyad, Oman

Arrow indicates flow direction in case of flooding

Summary & Conclusions

- The reduced dimension approach, described by the SWE, is extended to account for transport of suspended load and bedload.
- The multiphysics coupling appears, when the water depth is changing due to net bedload production or re-mobilization
- The test example of flow and sediment transport past an obstacle produces physically reasonable results

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Outlook

International Symposium
on Flash Floods
in Wadi Systems
ISFF

ISFF4

December 4-6, 2018

Casablanca, Morocco