COMSOL CONFERENCE
2018 LAUSANNE

DIPARTIMENTO INGEGNERIA dell'Energia, dei Sistemi, del Territorio e delle Costruzioni

UNIVERSITÀ DI PISA

Mazars' damage model for masonry structures: a case study of a church in Italy

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Seismic assessment of historical buildings

Goal:

- Safeguard of integrity and conservation

Challenges:

- Lack of knowledge of internal structure of walls, arches and vaults
- Huge variety of building techniques throughout history
- Strong nonlinear behavior

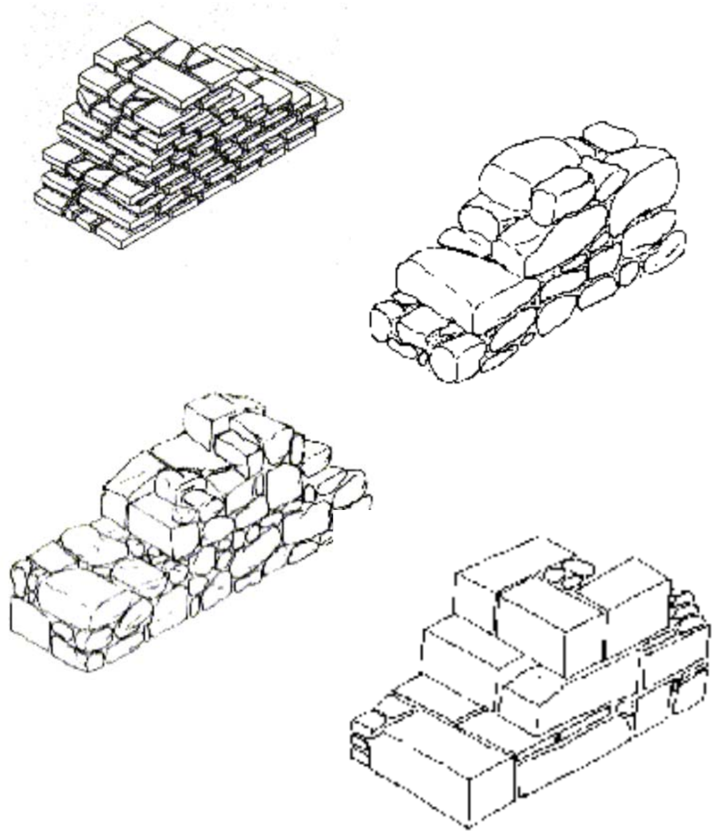
Structural modeling

Material properties: **Masonry**

- Low to negligible tensile strength
- Development of cracks
- Possibly high compressive strength

Modeling challenges

- Strongly nonlinear material
- Asymmetric behavior
- History dependent
- Variation of stiffness and dynamic properties



Images from: A. Bernardini, La vulnerabilità degli edifici: valutazione a scala nazionale della vulnerabilità sismica degli edifici ordinari CNR-Gruppo Nazionale per la Difesa dai Terremoti - Roma, 2000, 175 pp

Modeling approaches for cracking

Geometric approach

The crack and its evolution are defined by geometric entities

- Meshfree methods
- Adaptive BEM/FEM
- Lattice methods
- Particle methods

Non-geometric approach

The crack is introduced in local material properties

- Constitutive methods:
  - Continuum Damage Method (CDM)
  - Element Extinction Method, …
- Kinematic methods:
  - Enriched FEM,
  - XFEM, …

Mazars' damage model (CDM)

J. Mazars, G. Pijaudier-Cabot, Continuum damage theory - application to concrete, J. of Eng. Mech., ASCE, 115(2), 345–365 (1989).

- Damage variable *d:*

$$E^d = E_0 \cdot (1 - d)$$

$$d = \alpha_t d_t + \alpha_c d_c$$

E^d = Damaged Young's modulus

- Weighting coefficients:

$$\alpha_t = \sum_{i=1}^{3} \left(\frac{\langle \varepsilon_i^t \rangle \langle \varepsilon_i \rangle}{\tilde{\varepsilon}^2} \right)^{\beta} \quad \text{and} \quad \alpha_c = \sum_{i=1}^{3} \left(\frac{\langle \varepsilon_i^c \rangle \langle \varepsilon_i \rangle}{\tilde{\varepsilon}^2} \right)^{\beta} ,$$

$$\tilde{\varepsilon} = \sqrt{\sum\nolimits_{i=1}^{3} (\langle \varepsilon_i \rangle_+)^2}$$

- Tensile and compressive damage:

$$d_t(\kappa) = 1 - \frac{\kappa_0 (1 - A_t)}{\kappa} - A_t e^{-B_t(\kappa - \kappa_0)}$$

$$d_c(\kappa) = 1 - \frac{\kappa_0 (1 - A_c)}{\kappa} - A_c e^{-B_c(\kappa - \kappa_0)}$$

$\boldsymbol{\kappa} = \max(\kappa_0, \tilde{\varepsilon})$: State variable to store the maximum tensile strain.

5 material parameters: A_c, B_c, A_t, B_t, κ_0

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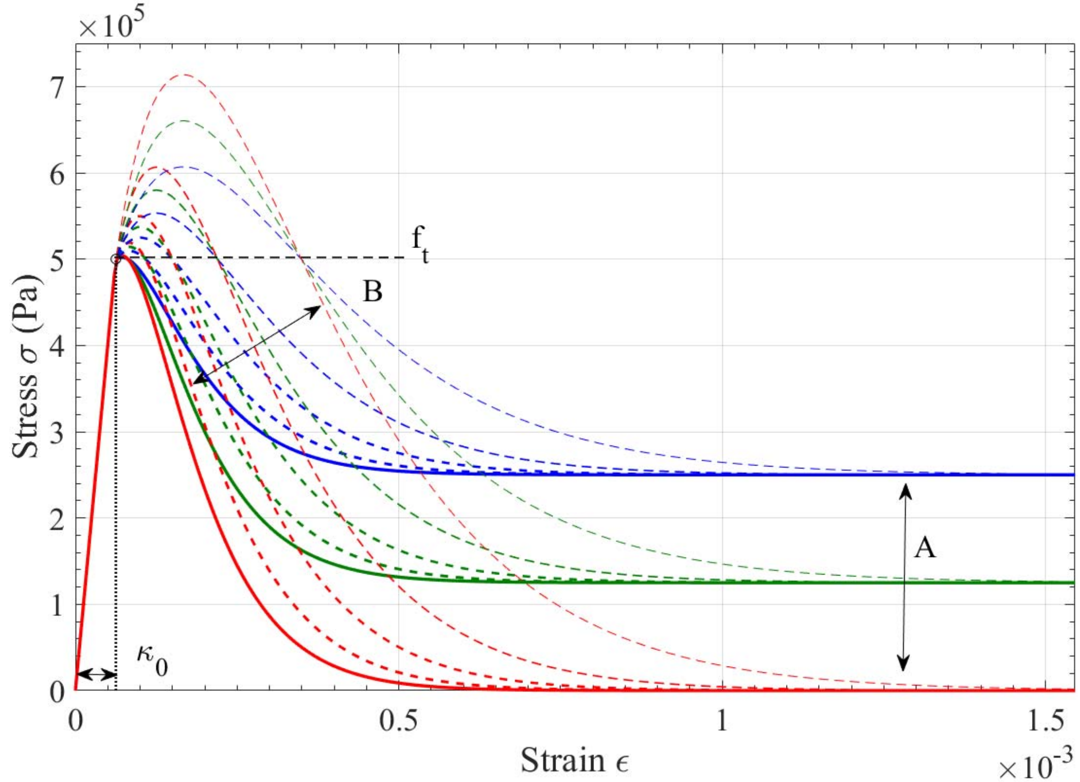


- $\boldsymbol{\kappa_0}$: Initial damage threshold, can be defined as a function of maximum tensile strength f_t as:

$$\kappa_0 = \frac{f_t}{E_0}$$

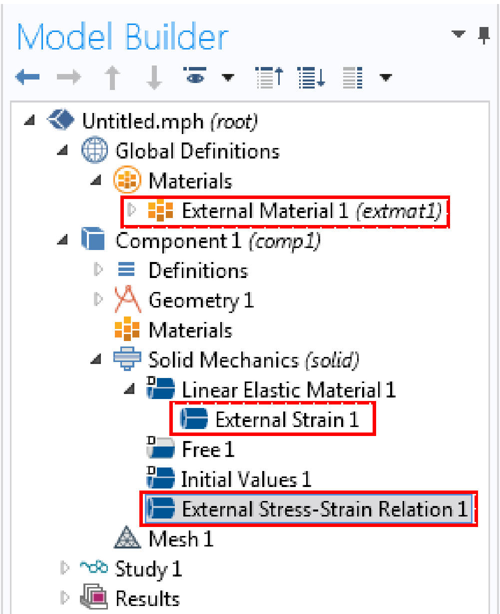
- $\boldsymbol{A_c, A_t}$: Residual strength ratio to peak strength

- $\boldsymbol{B_c, B_t}$: peak strength and softening branch steepness

Mazars' model: COMSOL Implementation

COMSOL Functionality: Structural Mechanics module,

External Stress-Strain Relation (DLL) written in C code

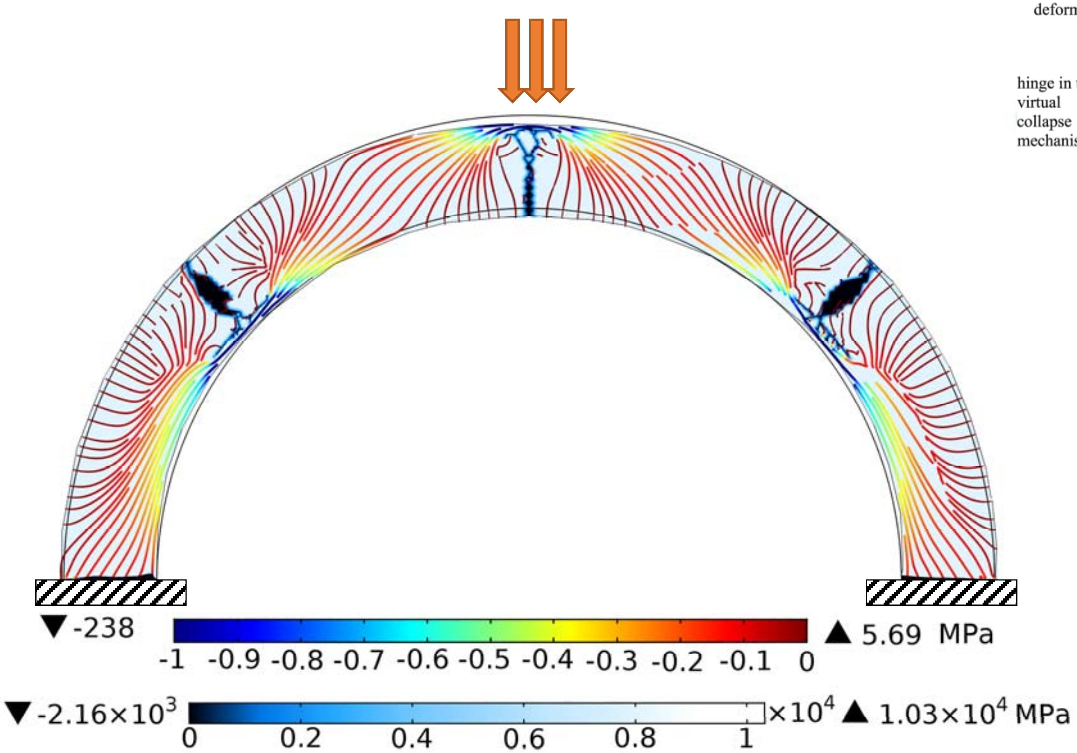



```
…
EXPORT int eval(double e[6],        // Input: Green-Lagrange strain
                                       tensor components in Voigt order
                                       (xx,yy,zz,yz,zx,xy)
                double s[6],        // Output: Second Piola-Kirchhoff
                                       stress components in Voigt order
                                       (xx,yy,zz,yz,zx,xy)
                double D[6][6],     // Output: Jacobian of stress with
                                       respect to strain, 6-by-6 matrix
                                       in row-major order
                int *nPar,          // Input: Number of material model
                                       parameters, scalar
                double *par,        // Input: Parameters: par[0] = E0,
                                       par[1] = nu0, ...
                int *nStates,       // Input: Number of states, scalar
                double *states)     // States, nStates-vector
…
           states[0] = eef;
           states[1] = damage;
…
```

Extract from COMSOL Application Library

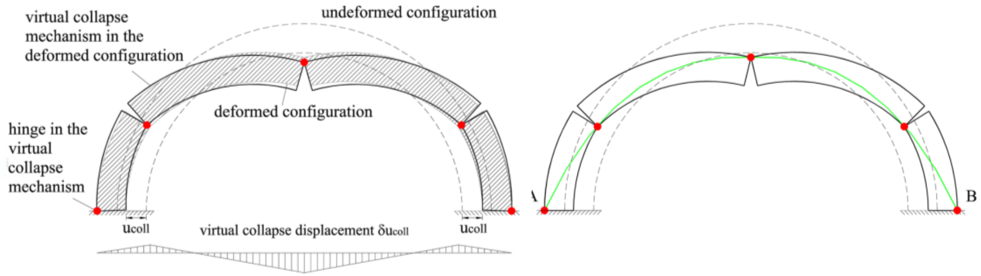
2D Test case

- Basic arch model loaded at keystone:



Plot: Third principal stress (compression) streamlines and equivalent Young modulus $E^d = E_0\ (1\text{-}d)$

From: "Collapse displacements for a mechanism of spreading-induced supports in a masonry arch", Simona Coccia, Fabio Di Carlo, Zila Rinaldi, Department of Civil Engineering, University of Rome ''Tor Vergata'', Rome, Italy - DOI 10.1007/s40091-015-0101-x



Modeling strategies:

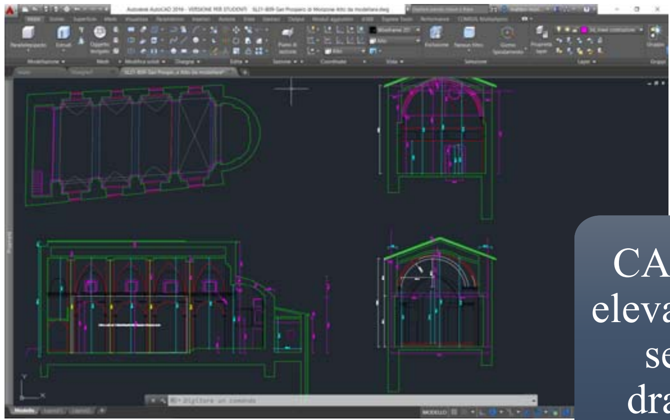
- Gradual ramping of external load with *auxiliary sweep* functionality
- Jacobian update on every iteration
- *Global equation* with auxiliary variable to achieve displacement controlled load increment

Results:

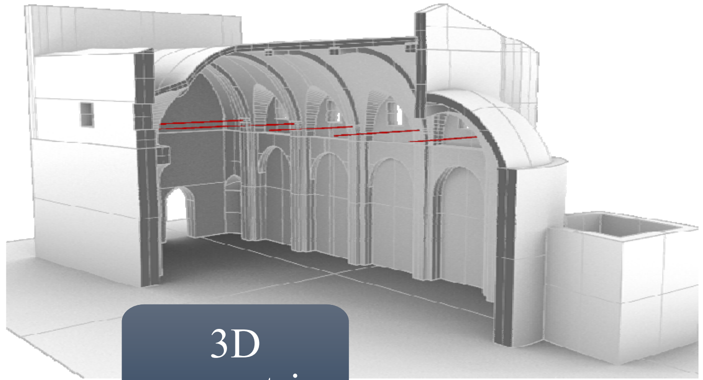
- Damage initiation at keystone at the intrados
- Successive Damage at quarters
- 5-hinge collapse mechanism

Case study: San Prospero di Monzone church

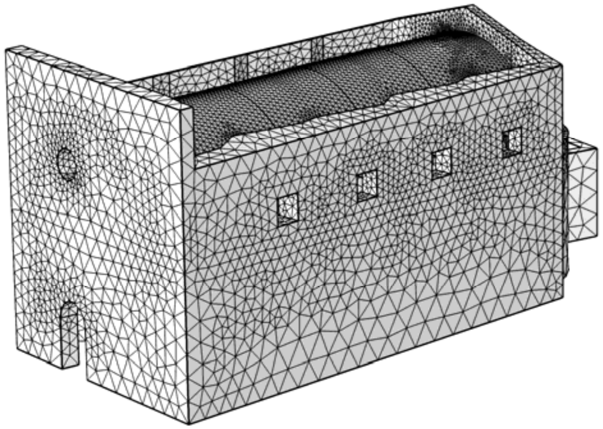
Site photography and measurements

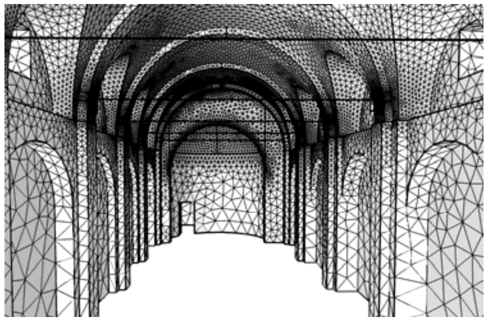


CAD plan, elevation and section drawings



3D geometric model



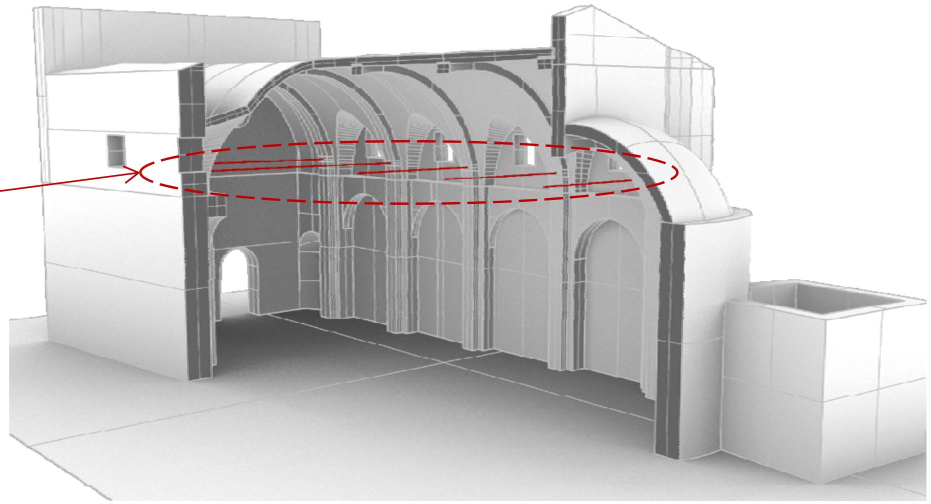


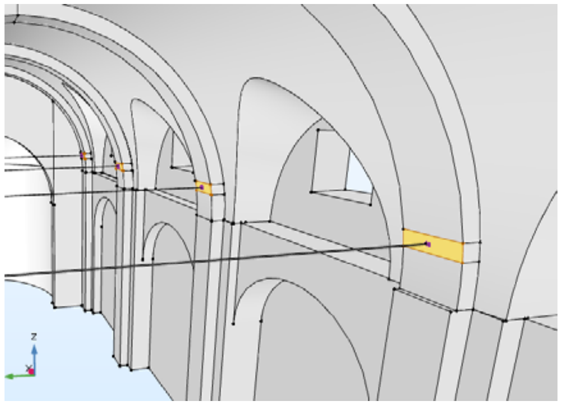
COMSOL 3D model and FE mesh

Static analysis under self weight load

3 scenarios:

- Elastic reference model
- Mazars' damage model without tie rods
- Mazars' damage model with tie rods



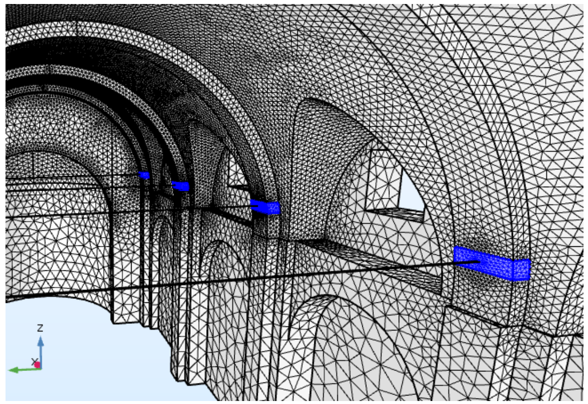


Tie rod modeling:

- Shell interface
- Solid-Shell Multiphysics coupling
- Elastic diffusion zone

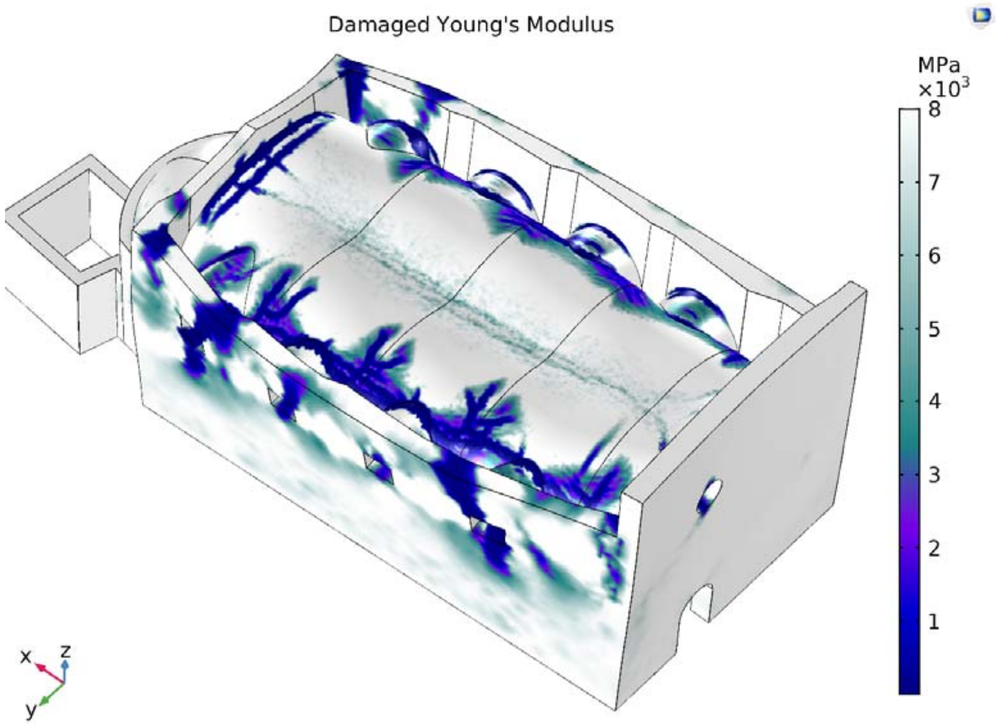
$$\mathbf{u}_{so} \approx \mathbf{u}_{sh} + z_l \mathbf{a}_{sh} \quad \text{on} \quad \partial\Omega_{so}$$

$$\text{such that} \quad -\frac{d}{2} + z_{offset} < z_l < \frac{d}{2} + z_{offset}$$

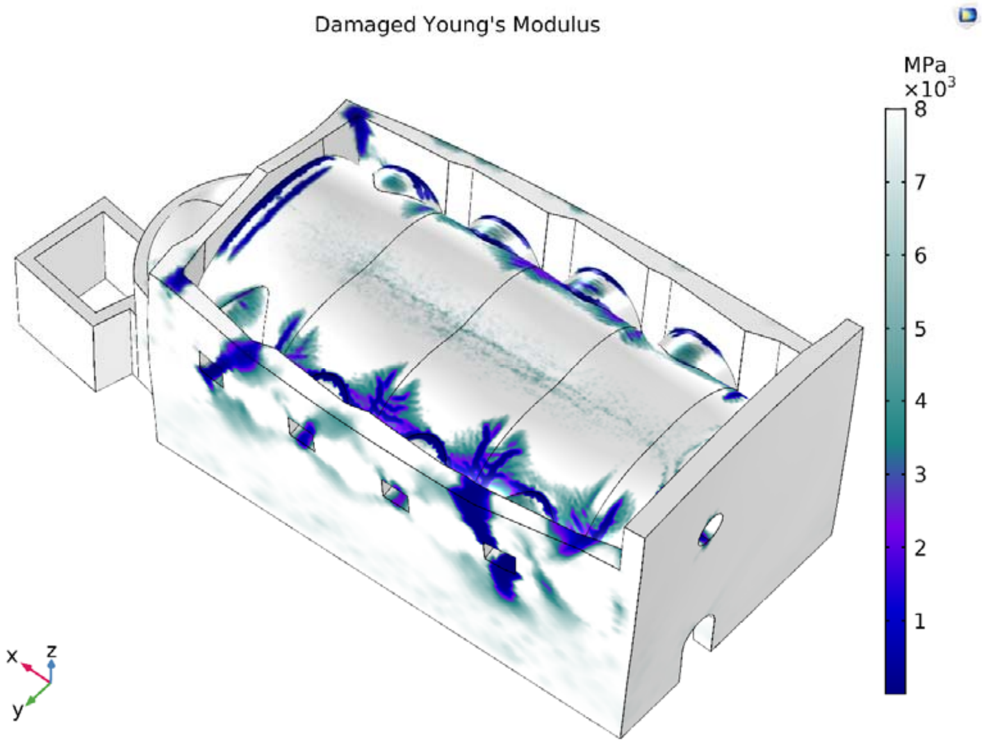


Static analysis under self weight load: results

Plot: `solid.dSde11*(nu0+1)*(2*nu0-1)/(nu0-1)`   Equivalent Young's Modulus = E^d



- Without tie rods



- With tie rods

Static analysis under self weight load: results

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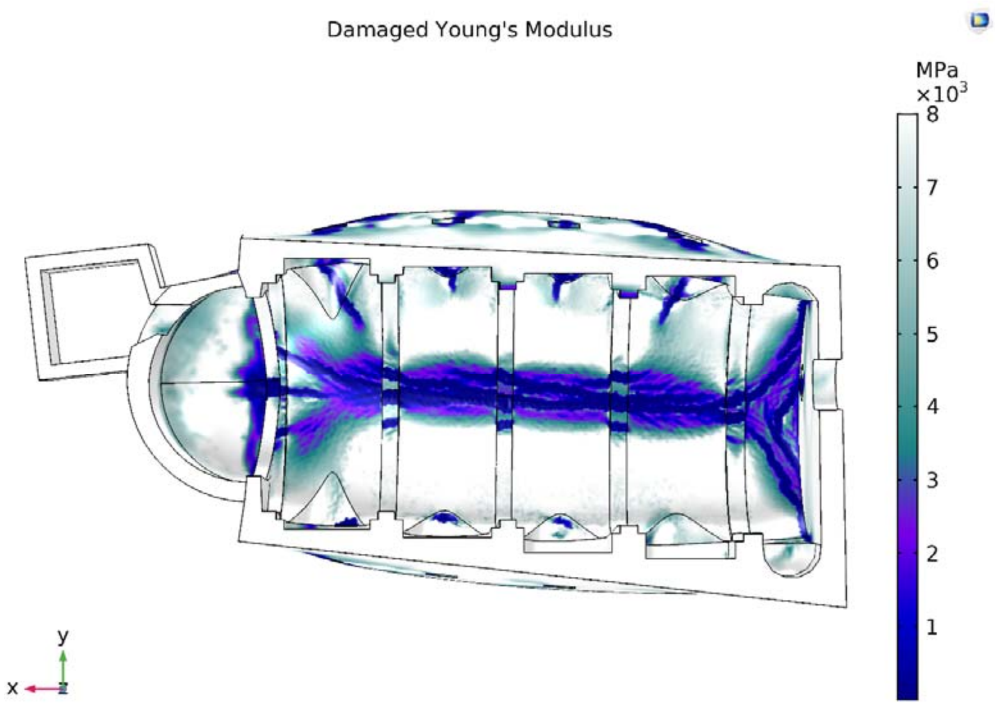
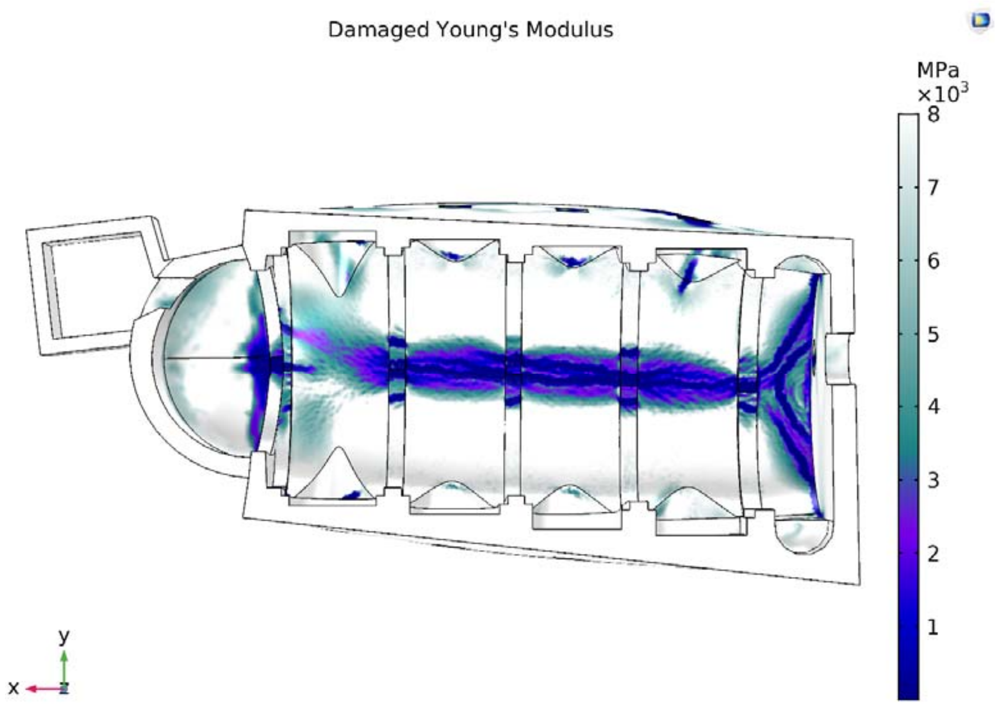


- Without tie rods

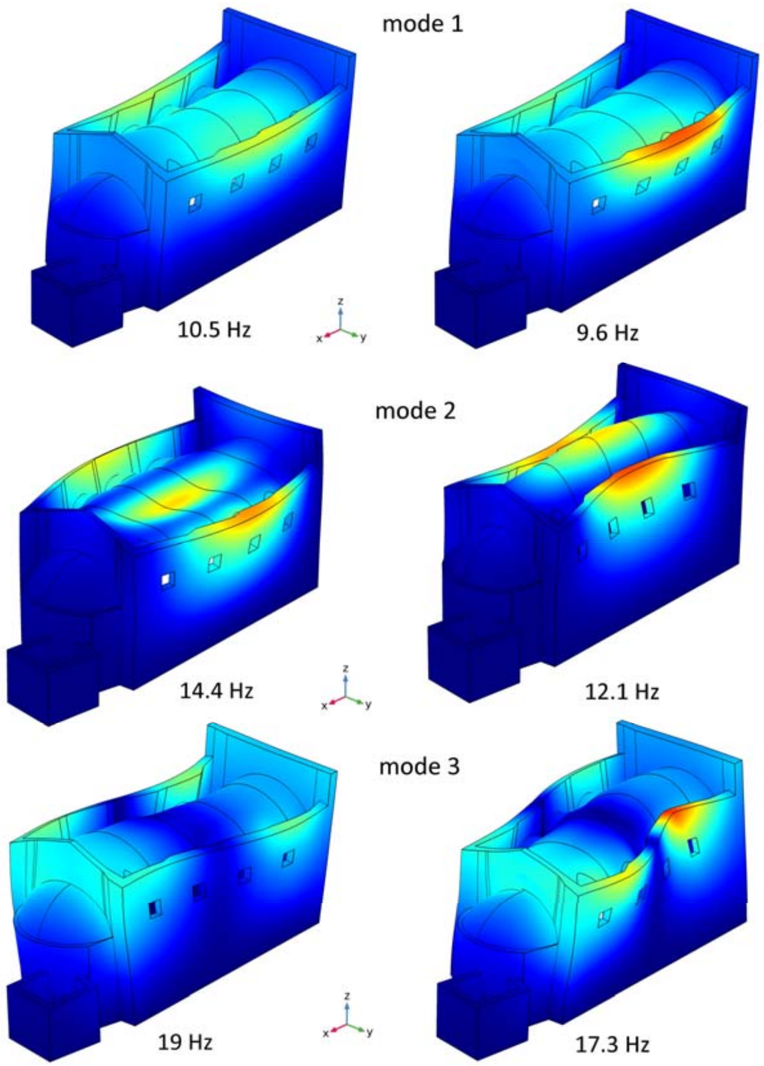


- With tie rods

Modal analysis

Solver linearization point: Computation of eigenmodes considering tangent stiffness: $E^d = E_0 \cdot (1\text{-}d)$

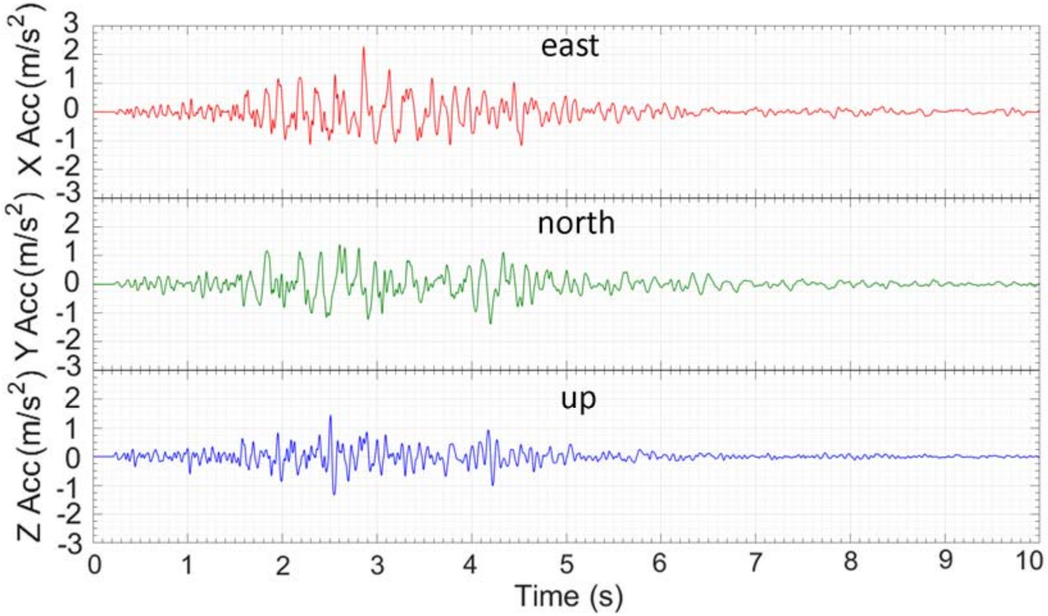
Elastic	Mazars damage model	Mazars damage model + tie rods
10.5	8.8	9.6
14.5	10.3	12.1
19. 0	16.1	17.3
21.5	18.1	19.8
24.0	18.9	20.4
25.8	19.5	21.9



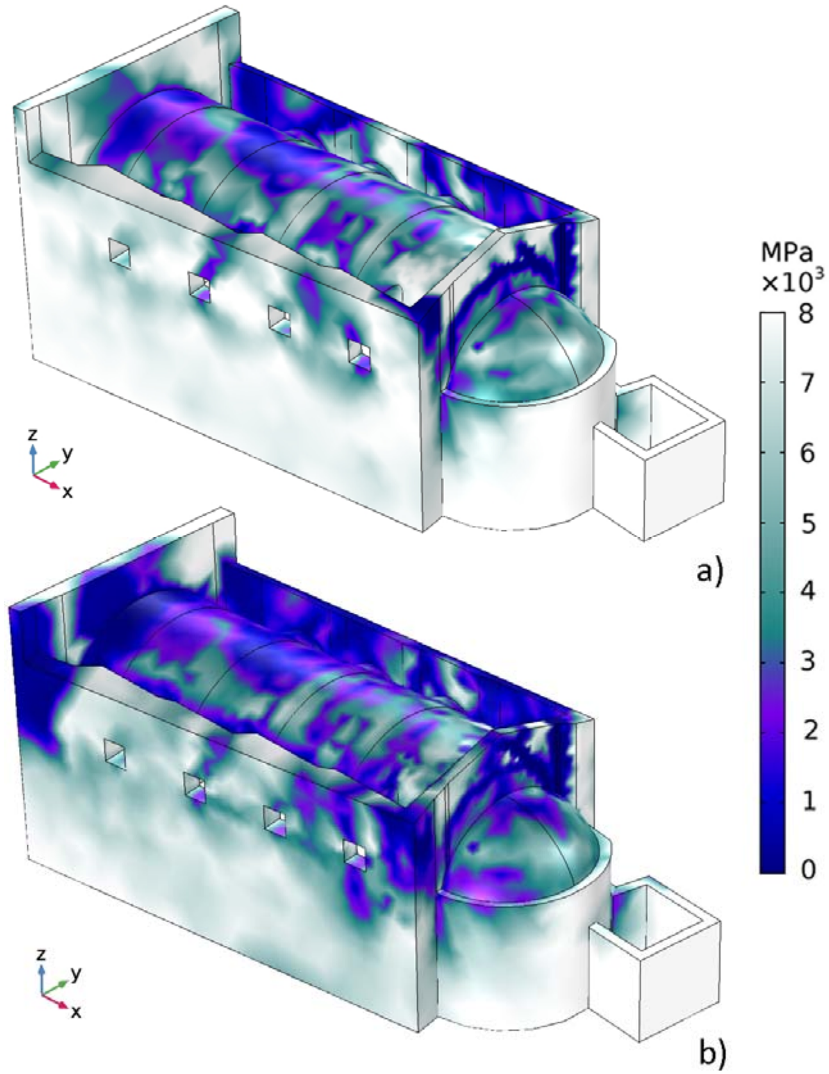
First three eigenmodes deformed shapes, tie rods model, elastic case (left) and Mazars (right).

Transient analysis
Tie rods model

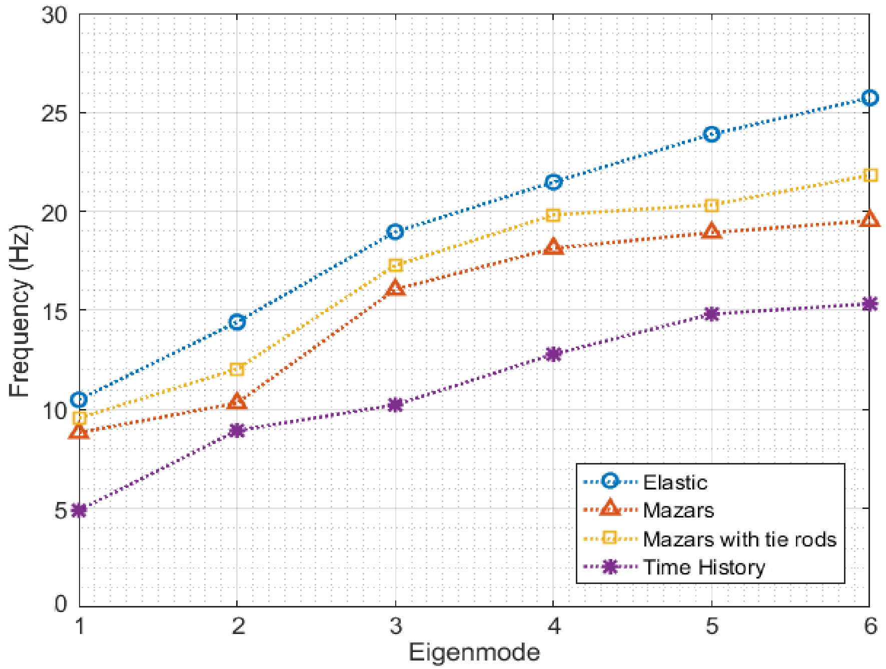
Recorded seismic event from National Accelerometric Network, Fivizzano station (FVZ) on 21 June 2013 (main shock at UTC 10:33).



Damage development from the vaulted structures and at connection between facade and ceiling and triumphal arch.

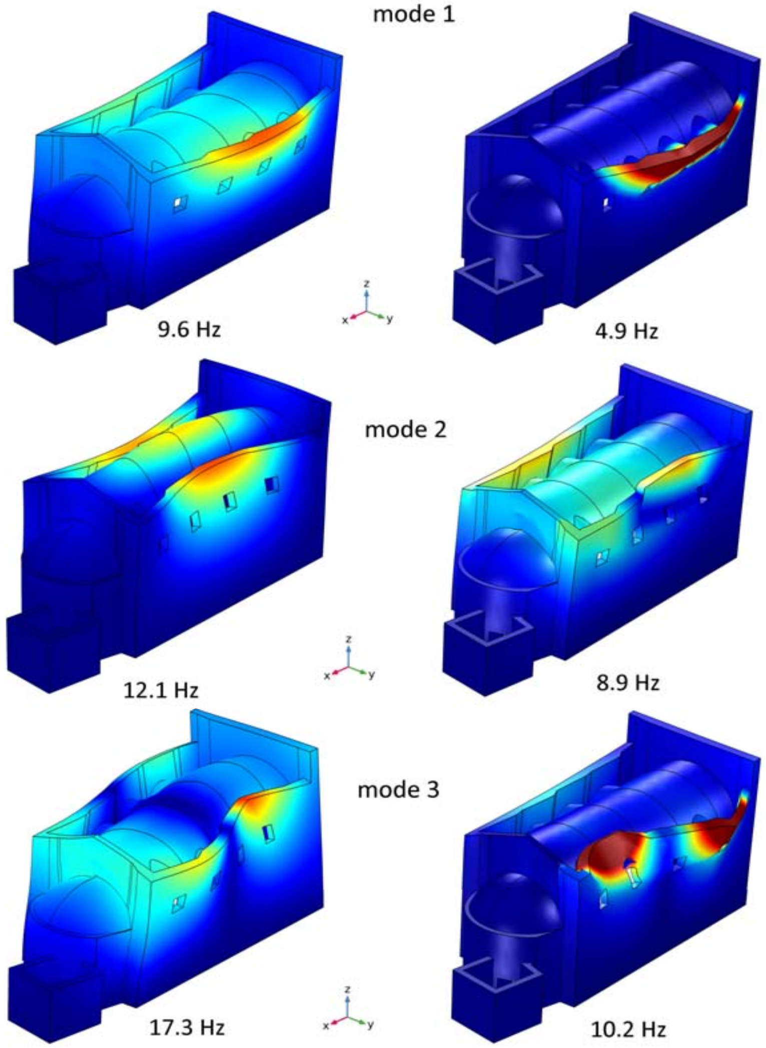


Post-transient modal analysis
Tie rod model



Observations:

- Strong frequency shift due to additional damage
- Variation in mode shapes
  - 1° mode: Localization of deformation
  - 2° mode: Three quarter wavelength along walls



First three eigenmodes deformed shapes, self-weight model (left) and post time history (right).

Conclusions:

- 3D FE Model of a masonry structure
- Adoption of Mazars' damage model via COMSOL's external material functionality
- Evaluation of structural damage
- Evaluation of damage influence on dynamic properties

Considerations:

- Mazars' model successfully captures damage from monotonic load histories
- Underestimation of structural resources for cyclic loads (crack closure effects)

Further developments:

- Implementation of Mazars μ-model for cyclic loads
- Fine tuning of material parameters through accurate fracture toughness considerations and fracture energy evaluation

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