Laminar Forced Convection Heat Transfer from Two Heated Square Cylinders in a Bingham Plastic Fluid

E. Tejaswini[1*], B. Sreenivasulu[2], B. Srinivas[3]

[1,2,3] Gayatri Vidya Parishad College of Engineering

[*]Corresponding Author: GVPCOE, Madhurawada, Visakhapatnam, Andhra Pradesh, 530048. E-mail address: teja.imemyself@gmail.com

Abstract: In this work, the momentum and heat transfer characteristics of two heated cylinders of square cross-section immersed in a streaming Bingham plastic medium have been studied. The study is done using COMSOL Multi-physics simulation software using the fluid flow and heat transfer modules. The governing differential equations (continuity, momentum and thermal energy) have been solved numerically over wide range of conditions as: plastic Reynolds number, $0.1 \leq Re \leq 40$, Prandtl number, $1 \leq Pr \leq 100$, Bingham number, $0 \leq Bn \leq 10$ and Distance between the two heated cylinders, S. Over this range of conditions, the flow is expected to be symmetric and steady. The detailed flow and temperature fields in the vicinity of the cylindrical surfaces are examined in terms of streamline and isotherm profiles respectively. The Nusselt number shows a positive dependence on the both Reynolds and Prandtl numbers. It is observed that the average Nusselt number increases with increasing Bingham number. Simulation results are validated with the available literature.

Keywords: Bingham plastic, Reynolds number, Prandtl number, Bingham number, Nusselt number

1. Introduction

Numerous studies concerning the flow and heat transfer from a square cylinder submerged in Newtonian fluids have been documented in the literature over the past years. While most of these studies have used air as the working fluid, suffice it to say that reliable values of Nusselt number for a square cylinder spanning the forced, mixed and free convection regimes are now available in the literature. In contrast, the corresponding studies entailing the flow of power-law fluids are of a more recent vintage. Heat transfer from a heated square cylinder to power-law fluids has been investigated in the steady flow regime, forced convection regime, mixed convection and most recently in the free convection regime. However, most of these studies are restricted to the so-called laminar flow regime, up to about Reynolds number values of 160-170 in the case of forced convection regime and up to about Grashof number values of 10^5 in the free convection regime. Finally, based on the numerical results, reliable expressions are available in the literature for estimating the value of Nusselt number in a new application in the two-dimensional laminar flow regime over wide range of power-law index and Prandtl number.

In contrast, much less is known about the analogous results in Bingham plastic fluids. While the currently available studies relating to the fluid mechanical aspects of a single square cylinder have been reviewed by Chhabra et al. The heat transfer from a single heated square cylinder in a Bingham plastic fluid in the forced convection regime have been investigated by Chhabra et al. using the finite element method. The analysis suggested that the use of an effective viscosity leads to reconciliation of Nusselt numbers over wide ranges of Bingham number. The creeping flow of Bingham plastic past an unconfined square cylinder by Chhabra et al. revealed the existence of three types of unyielded zones in the prevailing stress levels were below the threshold yield stress of the fluid. Admittedly a few numerical studies are available on free convection heat transfer in Bingham fluids in confined geometry like in enclosures with differentially heated walls and in ducts of various cross-sections, little is known about the heat transfer in Bingham fluids in external flows like that from a square cylinder.

Based on the foregoing description, it is thus fair to say that no prior results exist on the Nusselt number characteristics of a two square cylinders in Bingham plastic fluids at finite Reynolds numbers. This work aims to fill this gap in the current literature. In particular, the field equations (continuity, momentum and energy) are solved numerically for the following ranges of conditions: plastic Reynolds number, $0.1 \leq Re \leq 40$, Prandtl number, $1 \leq Pr \leq 100$, Bingham number, $0 \leq Bn \leq 10$ and Distance between the two heated cylinders, S. This work endeavours to elucidate the influence of each of these parameters on the momentum and heat transfer characteristics of two heated 2-D bars of square cross-section.

2. Problem Statement and Formulation

Two square cylinders of side length d, and heated to a temperature T_w are exposed to a free stream of Bingham fluid at temperature T_0 ($<T_w$) as shown schematically in Figure 1. The right wall is considered to be adiabatic and a no slip boundary to minimize the wall effect. While no information exists about the flow regimes in Bingham plastic fluids, by analogy with the transitions observed in Newtonian and power-law fluids, over the range of Reynolds number spanned here, the flow is expected to be steady and symmetric about the mid-plane and therefore computations have been carried out only in half domain and Figure 1 represents the symmetric computational domain.

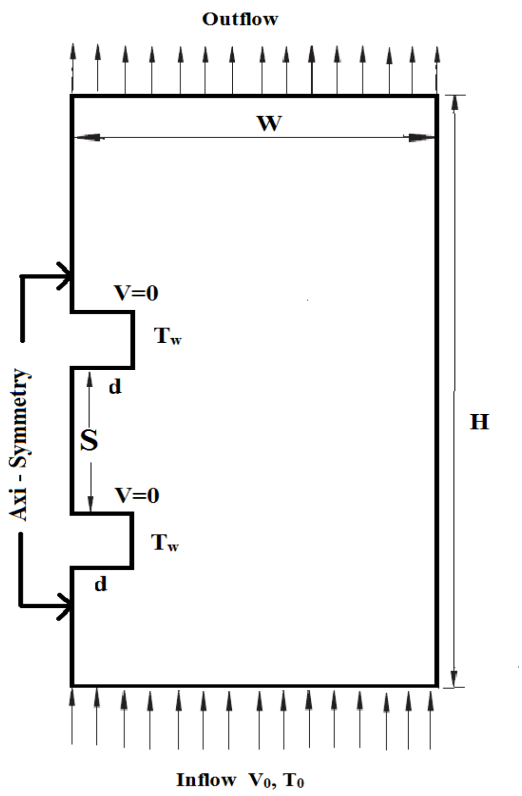


Figure 1. Schematics of flow and of computational domain.

The thermo-physical properties of the fluid (thermal conductivity, *k*, heat capacity, *C*, plastic viscosity, μ_B, yield stress, and density, ρ) are assumed to be temperature independent. The viscous dissipation effect is also neglected. While these two assumptions lead to the decoupling of velocity and temperature fields, but at the same time the first assumption also restricts the applicability of the results reported herein to situations where the maximum temperature difference $\Delta T = T_w - T_0$ is small. For the 2-D incompressible, steady, and laminar flow, the equations of continuity, momentum and thermal energy can be written in their dimensionless form as follows:

Continuity equation:

$$\nabla . V = 0$$

Momentum equation:

$$V.\nabla V = -p + \frac{1}{Re}\nabla . \tau$$

Thermal energy equation:

$$V.\nabla \theta = \frac{1}{RePr}\nabla^2 \theta$$

For a Bingham plastic fluid, the deviatoric part of the total stress tensor is given by constitutive relation which for a simple shear flow can be written in non-dimensional form as follows:

$$\tau = \left(1 + \frac{Bn}{|\dot{\gamma}|}\right)\dot{\gamma}, if\ |\tau| > Bn$$

$$\dot{\gamma} = 0, if\ |\tau| < Bn$$

The momentum and heat transfer characteristics are characterized by three dimensionless parameters, namely Bingham number, Reynolds number and Prandtl number appearing in the governing differential equations. For Bingham plastic fluids these are defined as follows:

Bingham number: It is the ratio of the yield stress (τ_0) to viscous stress ($\mu_B V_0/d$). Naturally, large values of Bingham number indicate strong yield stress effects (i.e., more like plastic flow conditions) and unyielded regions dominate the flow thereby leading to steeper velocity and temperature gradients in the small yielded regions. In the present case, it is written as follows:

$$Bn = \frac{\tau_0 d}{\mu_B V_0}$$

Reynolds number: It is the ratio of inertial ($\rho V_0^2 d^2$) to viscous forces ($\mu_B V_0 d$) which leads to the following definition:

$$Re = \frac{\rho d V_0}{\mu_B}$$

Prandtl number: It is the ratio of the momentum diffusivity to thermal diffusivity given as follows:

$$Pr = \frac{C\mu_B}{k}$$

It is worth noting that in Bingham fluid flows, as the viscosity varies throughout the flow, an effective viscosity expressed as $\mu'_{eff} = \frac{\tau_0}{\gamma'} + \mu_B$ might be more representative of the viscous stress within the flow than the constant plastic viscosity μ_B. Therefore Reynolds, Prandtl and Bingham numbers could have been defined more appropriately by using μ'_{eff} instead of μ_B. Doing so, and estimating a characteristic shear rate using V_0 and d as (V_0/d), gives the following "effective" definitions of these parameters:

$$Bn^* = \frac{Bn}{Bn+1}, \qquad Re^* = \frac{Re}{Bn+1},$$
$$Pr^* = \mathrm{Pr}(Bn+1)$$

The physically realistic boundary conditions for this configuration are as follows:

- **At the inlet boundary:** The uniform velocity, V_0, normal to inlet boundary for velocity and constant inlet fluid temperature T_0, for temperature have been prescribed at the inlet plane as follows:
$$V_x = 1, V_y = 0 \; and \; \theta = 0$$
- **On the surface of the cylinders:** As it is a solid surface, the usual no-slip boundary condition for flow and constant temperature condition for heat transfer are prescribed, i.e.,
$$V_x = V_y = 0, \theta = 1$$
- **At the line of symmetry:** Over the range of conditions spanned here, the flow is supposed to be steady and symmetric about the vertical centreline of the cylinder, so only half domain has been used here for computations to economize on computational efforts. The symmetry boundary condition used here is given below:
$$\frac{\partial V_x}{\partial y} = 0, V_y = 0 \; and \; \frac{\partial \theta}{\partial y} = 0$$
- **At the outlet boundary:** A zero diffusion flux condition for all dependent variables is prescribed at the outlet. This condition is consistent with the fully-developed flow assumption and similar to the homogeneous Neumann condition. The gradients in the lateral direction can, however, still exist. Thus the following condition is prescribed here:
$$\frac{\partial \varphi}{\partial x} = 0 \; where \; \varphi = \left(V_x, V_y \; and \; \theta\right)$$

Nusselt number (Nu): This is the non-dimensional rate of heat transfer between the fluid and the cylinders. Since the no-slip condition is prescribed on the surface of the cylinders, it can readily be shown that the local value of the Nusselt number is given as follows:

$$Nu_L = \frac{hd}{k} = -\left(\frac{\partial \theta}{\partial n_s}\right)$$

where n_s is the outward drawn unit normal vector on the surface of the cylinder. As the local Nusselt number varies along the surface of the cylinders, from a practical standpoint, the overall mean value of the Nusselt number is required in process design calculations to estimate the rate of heat loss (or gain) from the cylinders. The surface averaged value is calculated simply by integrating the local values of the Nusselt number over the whole surface of the cylinders as:

$$Nu = \frac{1}{S}\int Nu_L dS$$

3. Numerical Methodology

The governing differential equations subject to the aforementioned boundary conditions have been solved numerically using the finite element based solver COMSOL Multiphysics (Version 4.3a) for both meshing the computational domain as well as to map the flow domain in terms of the primitive variables *u-v-p*. Since gradients are expected to be steep near the cylinder surfaces as well as near as near the interface between the rigid (unyielded) and fluid (yielded) zones, a fine mesh is required in these regions. In this study a triangular cells have been used to mesh these regions of the computational domain. Furthermore, the steady, 2-D, laminar flow module and heat transfer module are used with PARDISO solver with parametric sweep to solve the equations.

4. Results and Discussion

In this study, extensive numerical results have been obtained over wide range of dimensionless parameters as ($0.1 \le Re \le 40$; $1 \le Pr \le 100$; $1 \le Bn \le 10$). The influence of these parameters on streamline patterns, isotherm contours, local Nusselt number and average Nusselt number have been investigated in detail. However, prior to undertaking a detailed presentation of the new results, it is desirable to establish the accuracy and reliability of the present results.

4.1 Validation of Results

As noted above, in order to ascertain the accuracy and reliability of the present solver, the present numerical results (for Bn=0) are compared with the literature values for Newtonian liquids. Table 1 represents the values of the average Nusselt number at Prandtl number values of Pr=1 and Pr=100 at different Reynolds numbers and the results are seen to be in a good agreement with the previous studies. Suffice it to say that the present results reported are considered to be reliable to within 2-2.5%.

Table 1: Comparison of average Nusselt number with literature in Newtonian media (Bn=0)

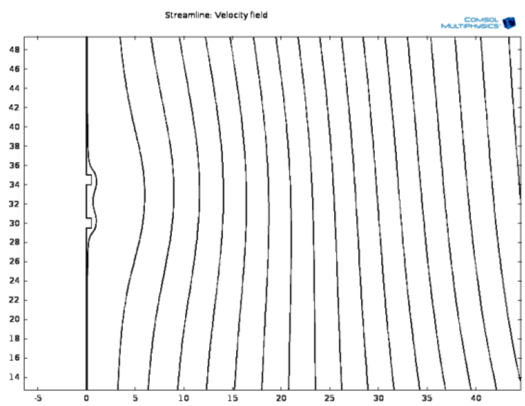
Re	Average Nusselt number, Nu						
	Pr=1			Pr=100			
	Dhiman et al.	Chhabra et al.	Present	Dhiman et al.	Rao et al.	Chhabra et al.	Present
0.1	-	0.4328	0.4198	-	0.5599	0.5581	0.5576
5	1.3442	1.3021	1.3096	5.5044	5.3836	5.5276	5.3897
10	1.7534	1.7531	1.7138	7.4218	7.3324	7.4521	7.3825
20	2.3055	2.3230	2.2569	10.398	10.4904	10.381	10.459
30	2.7042	2.7301	2.6522	12.513	-	12.507	12.501

4.2 Streamline Contours

It is customary to visualize the general features of the flow in terms of streamline patterns. Representative streamlines are presented in Figure 2 for a range of values of Reynolds number and Bingham number. Only some part of the computational domain is shown in figure 2 for better understanding of the streamline contours.

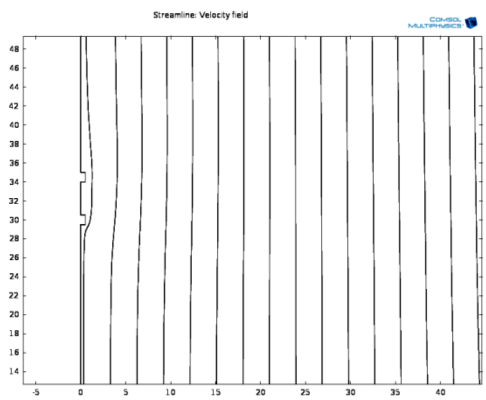
Bn=0

Re=0.1



Bn=0

Re=40



Bn=10

Bn=10

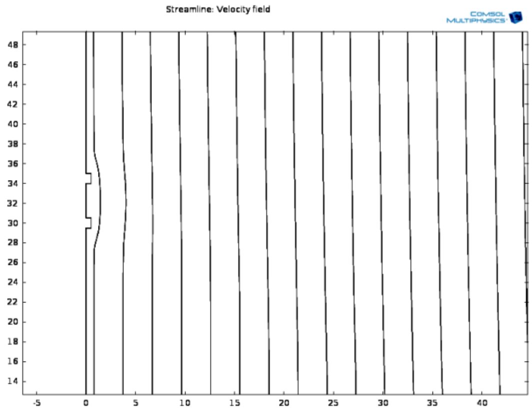


Figure 2a. Streamline patterns near the cylindrical surfaces for different Bingham numbers (a) Re=0.1.

Figure 2a shows the influence of Bingham number on the streamlines contour at Re=0.1 and it can be seen that at Bingham number, Bn=0 (Newtonian fluid) there is no flow separation. In the absence of any yield stress effects, the flow is governed by a balance between the viscous and pressure forcesat such low Reynolds numbers. On the other hand, as the value of the Reynolds number is increased, the

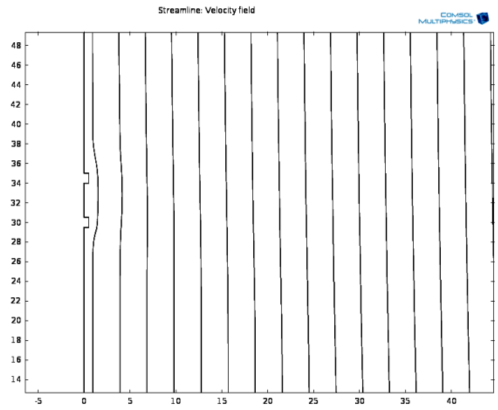


Figure 2b. Streamline patterns near the cylindrical surfaces for different Bingham numbers (b) Re=40.

flow pattern is now influenced by inertial, viscous, yield stress and pressure forces as shown in figure 2b.

4.3 Isotherm Contours

As noted earlier, the temperature field and Nusselt number show additional dependence on Prandtl number Pr and Pr^*. Representative isotherms elucidating the influence of Re and Pr on the temperature field are shown in figure 3.

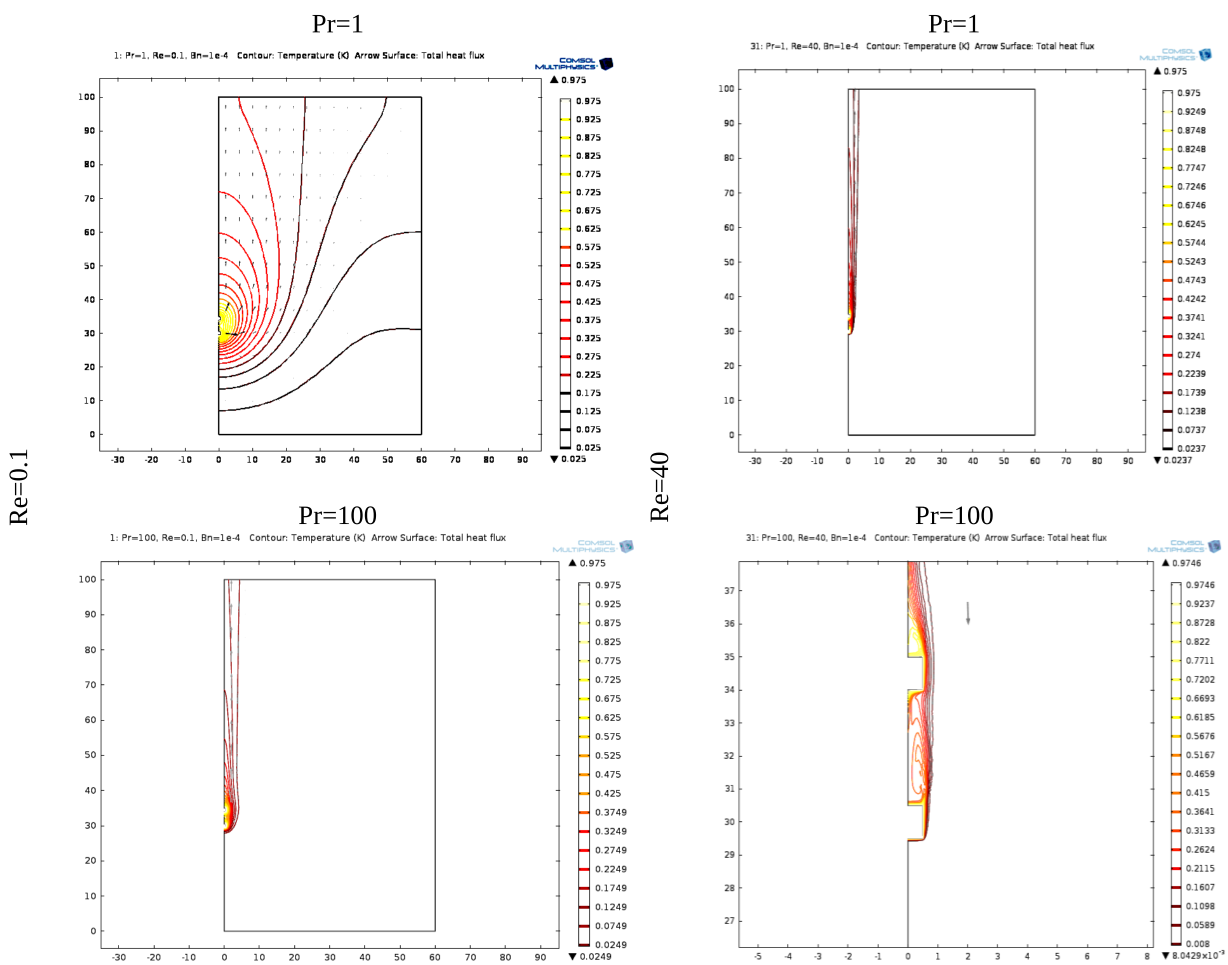


Figure 3a. Isotherm profiles near the cylindrical surfaces (a) Re=0.1.

A detailed examination of the isotherm contours suggests the following key trends. For a fixed value of Bingham number, at low Reynolds numbers when convection is weak and heat transfer is dominated by conduction, isotherms are seen to be more or less similar and this trend persists even at high Bingham numbers. This is not shown in the figure as there is not much variation of temperature with Bingham number. This observation is consistent with the fact that the unyielded regions grow with Bn and under these conditions also, the only possible mode of heat transfer is conduction. This is so especially at low values of Peclet number (Pe=Re*Pr, Pe=0.1), as seen in the figure. Some distortion of isotherms is seen at Pr=100 i.e., Pe=10. This distortion is due to the thinning of the thermal boundary layer at such high values of Prandtl numbers. With the increasing Reynolds

Figure 3b. Isotherm profiles near the cylindrical surfaces (a) Re=40.

number, the thinning of the thermal boundary layer is evident by the way of crowding of isotherms in generalon all sides of the cylinders. Since the thermal boundary layer progressively thins with the increasing Reynolds and Prandtl numbers, one would expect the Nusselt number to exhibit positive dependence on both these parameters.

4.4 Local Nusselt number on the surface of the lower cylinder

Typical variation of the local Nusselt number over the surface of the lower square cylinder is shown in Figure 4. The effect of heat transfer from the lower cylinder is more when compared to the second one because the lower cylinder is near to the inlet wall. As postulated in the previous section, the local Nusselt number (Nu_L) shows a positive dependence on Reynolds number (Re), Prandtl number (Pr) and Bingham number (Bn).

Pr=1 Pr=100

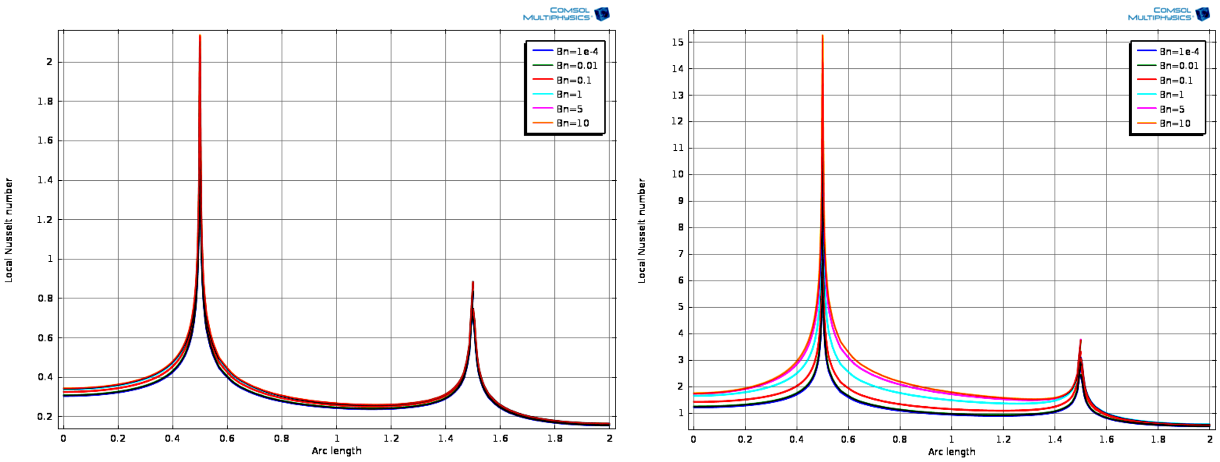


Figure 4a. Variation of Local Nusselt number along the surface of the lower cylinder at Re=0.1

Pr=1 Pr=100

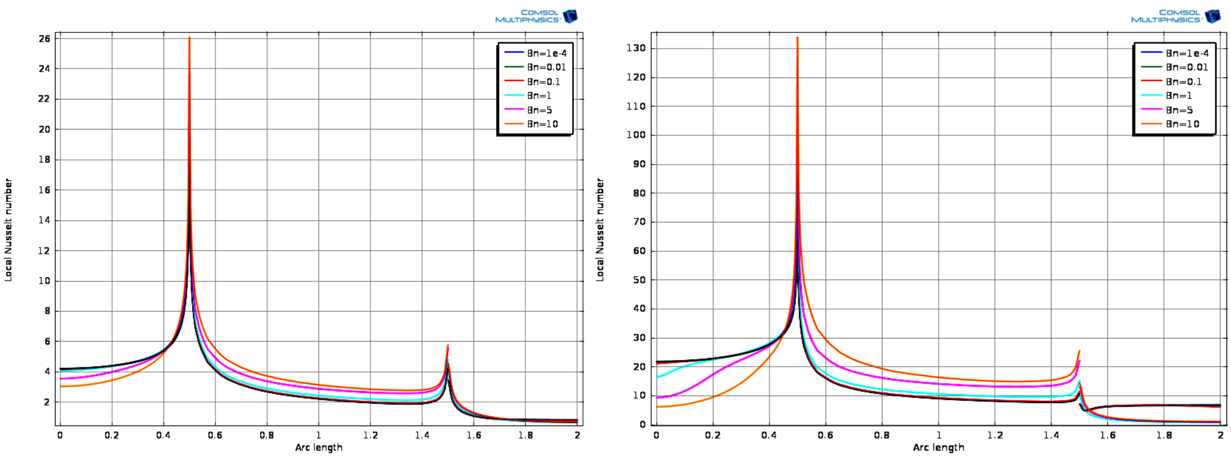


Figure 4b. Variation of Local Nusselt number along the surface of the lower cylinder at Re=40

The enhancement in heat transfer is due to the thinning of the thermal boundary layer with the increasing Reynolds and Prandtl numbers, albeit the effect of Reynolds number is stronger than that of Prandtl number which is consistent with the boundary layer analysis. The positive correlation between the heat transfer and Bingham number, on the other hand, stems from the diminishing yielded regions which in turn result in steep temperature gradients in the proximity of the heated cylinder.

5. Conclusions

The laminar forced convection heat transfer of a Bingham plastic fluid past two 2-D square cylinders has been studied numerically using the finite element method. Extensive results on streamline, isotherms, and Nusselt number have been obtained in order to elucidate the influence of the Reynolds number and Bingham number on the velocity an shear rate distributions in the close proximity of the cylinders. The temperature field and heat transfer characteristics show additional dependence on Prandtl number. The analysis of the present results suggests that the use of an effective viscosity leads to reconciliation of Nusselt number results over wide ranges of Bingham number. This work spans the following ranges of conditions: $0.1 \leq Re \leq 40$, $1 \leq Pr \leq 100$, $0 \leq Bn \leq 10$ when defined in terms of Bingham plastic viscosity.

6. References

1. N. Nirmalkar, R.P. Chhabra, R.J. Poole, Laminar forced convection heat transfer from a heated square cylinder in a Bingham plastic fluid, Int. J. Heat and Mass Transfer 56. (2013) 625-639
2. N. Nirmalkar, R.P. Chhabra, R.J. Poole, On creeping flow of a Bingham plastic fluid past a square cylinder, J. Non-Newt. Fluid Mech. 171–172 (2012) 17–30.
3. R.P. Chhabra, Fluid flow and heat transfer from circular and non-circular cylinders submerged in non-Newtonian liquids, Adv. Heat Transfer 43 (2011) 289–417.
4. A. Sharma, V. Eswaran, Effect of channel-confinement on the two-dimensional laminar flow and heat transfer across a square cylinder, Numer. Heat Transfer 47A (2005) 79–107.
5. A. Sharma, V. Eswaran, Effect of aiding and opposing buoyancy on the heat and fluid flow across a square cylinder at Re = 100, Numer. Heat Transfer 45A (2004) 601–624.
6. A. Sharma, V. Eswaran, Heat and fluid flow across a square cylinder in the two-dimensional laminar flow regime, Numer. Heat Transfer 45A (2004) 247–269.
7. S. Sen, S. Mittal, G. Biswas, Flow past a square cylinder at low Reynolds numbers, Int. J. Numer. Methods Fluids 67 (2011) 1160–1174.
8. S. Turki, H. Abbassi, S.B. Nasrallah, Two-dimensional laminar fluid flow and heat transfer in a channel with a built-in heated square cylinder, Int. J. Therm. Sci. 42 (2003) 1105–1113.
9. K.S. Chang, C.J. Choi, Separated laminar natural convection above a horizontal isothermal square cylinder, Int. Comm. Heat Mass Transfer 13 (1986) 201–208.
10. A.K. Dhiman, R.P. Chhabra, V. Eswaran, Heat transfer to power-law fluids from a heated square cylinder, Numer. Heat Transfer 52A (2007) 185–201.
11. A.K. Dhiman, R.P. Chhabra, V. Eswaran, Steady flow across a confined square cylinder: effects of power-law index and blockage ratio, J. Non-Newt. Fluid Mech. 148 (2008) 141–150.
12. P.K. Rao, A.K. Sahu, R.P. Chhabra, Momentum and heat transfer from a square cylinder in power-law fluids, Int. J. Heat Mass Transfer 54 (2011) 390–403.
13. A.K. Dhiman, R.P. Chhabra, V. Eswaran, Steady mixed convection from a confined square cylinder, Int. Comm. Heat Mass Transfer 35 (2008) 47–55.
14. C. Sasmal, R.P. Chhabra, Laminar natural convection from a heated square cylinder immersed in power-law liquids, J. Non-Newt. Fluid Mech. 166 (2011)811–830.
15. R.B. Bird, G.C. Dai, B.J. Yarusso, The rheology and flow of viscoplastic materials, Rev. Chem. Eng. 1 (1983) 1–70.