Transformation Optics Simulation Method for Stimulated Brillouin Scattering

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Introduction: Stimulated Brillouin scattering (SBS) is the stimulated nonlinear interaction between two light waves and an elastic wave [1]. Recently, there have been efforts to investigate and model structured systems able to enhance SBS effects, e.g. amplification and slow light [2-4]. Such devices are commonly designed through numerical simulations. However, accounting for SBS effects in finite-element simulations is not straightforward, and *ad hoc* methods must be developed to implement this functionality. We propose a transformation optics [5-7]-based approach for the simulation of solid-state SBS in the frequency domain, and demonstrate its possibilities when applied to a simple bulk system and to a more complex waveguide geometry.

Computational Methods: Typically, frequency-domain simulations of electromagnetics and mechanics are cast in different frames of reference. Thus, the coupling of the two physics is a non-straightforward task. Time-domain simulations would be able to overcome this difficulty, but are impractical due to the tremendous difference in frequency between light and elastic waves. To correctly simulate these phenomena in the frequency domain, it is necessary to devise a means of accommodating the difference between the two frames of reference. Two coordinate systems are commonly employed in continuum mechanics: the Eulerian (or spatial, **x**) and the Lagrangian (or material, **X**), related by the displacement **u** and the displacement gradient *F*.

$$\mathbf{x} = \mathbf{X} + \mathbf{u}(\mathbf{X}, t)$$
$$d\mathbf{x} = \bar{\bar{F}}\, d\mathbf{X}$$
$$\bar{\bar{F}} = \bar{\bar{I}} + \nabla_{\mathbf{X}}\mathbf{u}$$
$$\det \bar{\bar{F}} = \frac{\rho_0}{\rho}$$

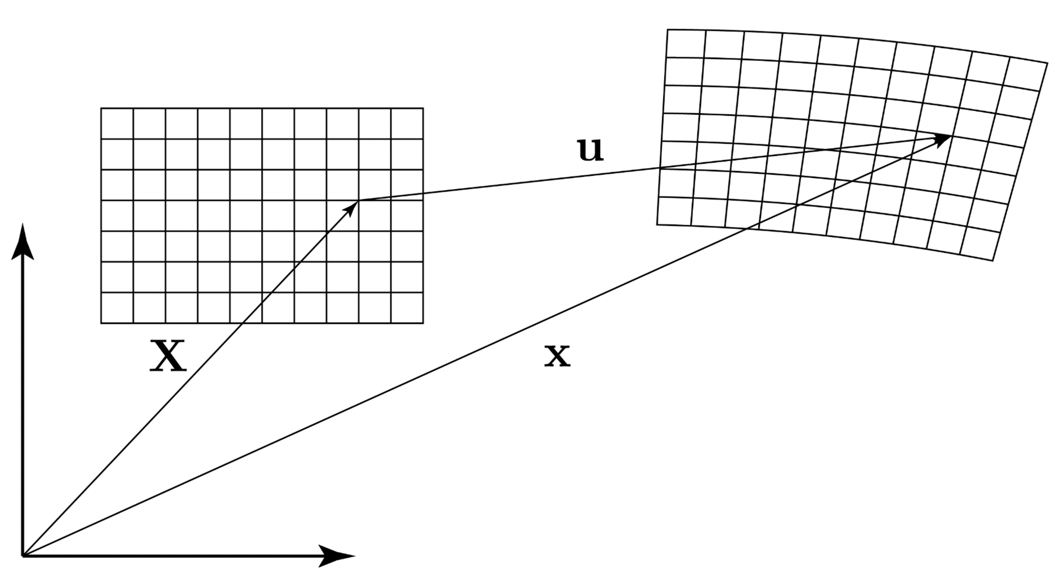

Figure 1. Relation between Lagrangian coordinates **X**, Eulerian coordinates **x**, and displacement **u**.

In transformation optics (TO), the material properties of a background medium are transformed to achieve a desired effect, represented by a coordinate transformation.

$$\bar{\bar{\epsilon}}' = \frac{\bar{\bar{A}}\bar{\bar{A}}^T}{\det \bar{\bar{A}}}\epsilon = \bar{\bar{g}}\epsilon$$
$$\bar{\bar{\mu}}' = \frac{\bar{\bar{A}}\bar{\bar{A}}^T}{\det \bar{\bar{A}}}\mu = \bar{\bar{g}}\mu,$$

where *A* is the Jacobian of the transformation. Here, we propose to use the displacement gradient *F* as the Jacobian of a transformation that will represent the frame movement by a variation of the material properties in space and time. Assuming the displacement field and density to oscillate at the frequency Ω, we obtain

$$\bar{\bar{g}} = \sum_{n=-3}^{3} \bar{\bar{g}}_n e^{in\Omega t}, \quad \bar{\bar{g}}_n = \bar{\bar{g}}^*_{-n}.$$

Starting with fields at frequencies ω_1 (pump) and ω_2 (signal), we state conservation of energy as $\omega_1 = \omega_2 + \Omega$ (Stokes SBS) and apply Maxwell's equations, including the TO metric and the nonlinear SBS term $\Delta\epsilon$ to obtain two mutually coupled wave-like equations:

$$\tilde{\mathbf{E}} = \mathrm{Re}\left(\mathbf{E}_1 e^{i\omega_1 t} + \mathbf{E}_2 e^{i\omega_2 t}\right) \qquad -\nabla \times \tilde{\mathbf{E}} = \frac{\partial}{\partial t}\left(\bar{\bar{g}}\mu\tilde{\mathbf{H}}\right)$$
$$\tilde{\mathbf{H}} = \mathrm{Re}\left(\mathbf{H}_1 e^{i\omega_1 t} + \mathbf{H}_2 e^{i\omega_2 t}\right) \qquad \nabla \times \tilde{\mathbf{H}} = \frac{\partial}{\partial t}\left[\bar{\bar{g}}\left(\epsilon + \epsilon_0\Delta\epsilon\right)\tilde{\mathbf{E}}\right] + \bar{\bar{g}}\sigma_e\tilde{\mathbf{E}},$$

$$\nabla \times \bar{\bar{A}}^{-1}\nabla \times \mathbf{E}_1 - \omega_1^2\bar{\bar{C}}\mathbf{E}_1 = \omega_1^2\left[\bar{\bar{K}}\mathbf{E}_1 + \left(\bar{\bar{D}} + \bar{\bar{L}}\right)\mathbf{E}_2\right] - i\omega_1\nabla \times \left(\bar{\bar{A}}^{-1}\bar{\bar{B}}\mathbf{H}_2\right)$$

$$\nabla \times \bar{\bar{A}}^{-1}\nabla \times \mathbf{E}_2 - \omega_2^2\bar{\bar{C}}\mathbf{E}_2 = \omega_2^2\left[\left(\bar{\bar{D}}^* + \bar{\bar{L}}^*\right)\mathbf{E}_1 + \bar{\bar{K}}\mathbf{E}_2\right] - i\omega_2\nabla \times \left(\bar{\bar{A}}^{-1}\bar{\bar{B}}^*\mathbf{H}_1\right),$$

where

$$\bar{\bar{A}} = \bar{\bar{g}}_0\mu \qquad \bar{\bar{B}} = \bar{\bar{g}}_1\mu \qquad \bar{\bar{C}} = \bar{\bar{g}}_0\,\epsilon \qquad \bar{\bar{D}} = \bar{\bar{g}}_1\epsilon$$
$$\bar{\bar{K}} = \epsilon_0\left(\bar{\bar{g}}_1\Delta\epsilon^* + \bar{\bar{g}}_1^*\Delta\epsilon\right)/2 \quad \bar{\bar{L}} = \epsilon_0\left(\bar{\bar{g}}_0\Delta\epsilon + \bar{\bar{g}}_2\Delta\epsilon^*\right)/2.$$

The mechanical aspect of the phenomenon is still described by Lagrangian equation of motion of continuum mechanics, driven by the optical forces **F** at the frequency Ω

$$-\rho_0\Omega^2\mathbf{u} = \nabla\cdot\bar{\bar{C}} : \nabla\mathbf{u} + \mathbf{F}.$$

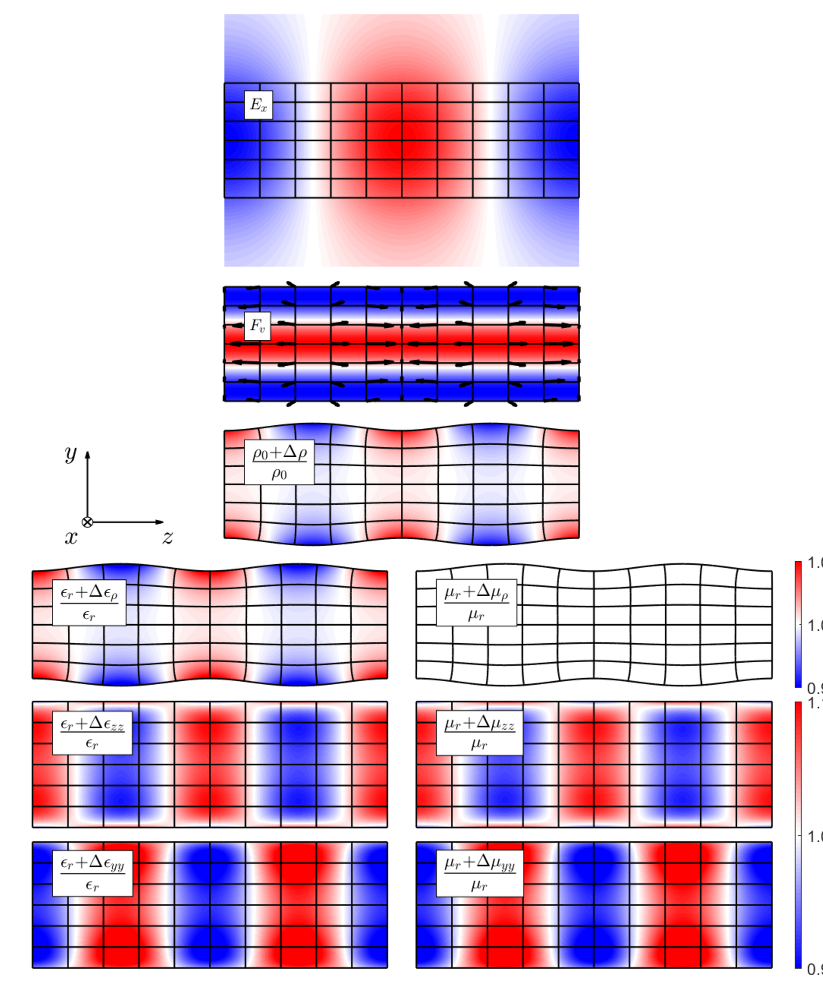

Figure 2. (Top 5 panels) optical fields and forces, elastic modes with corresponding variations in density and dielectric properties; (bottom 4 panels) transformed dielectric properties.

Results: **1D bulk amplifier**

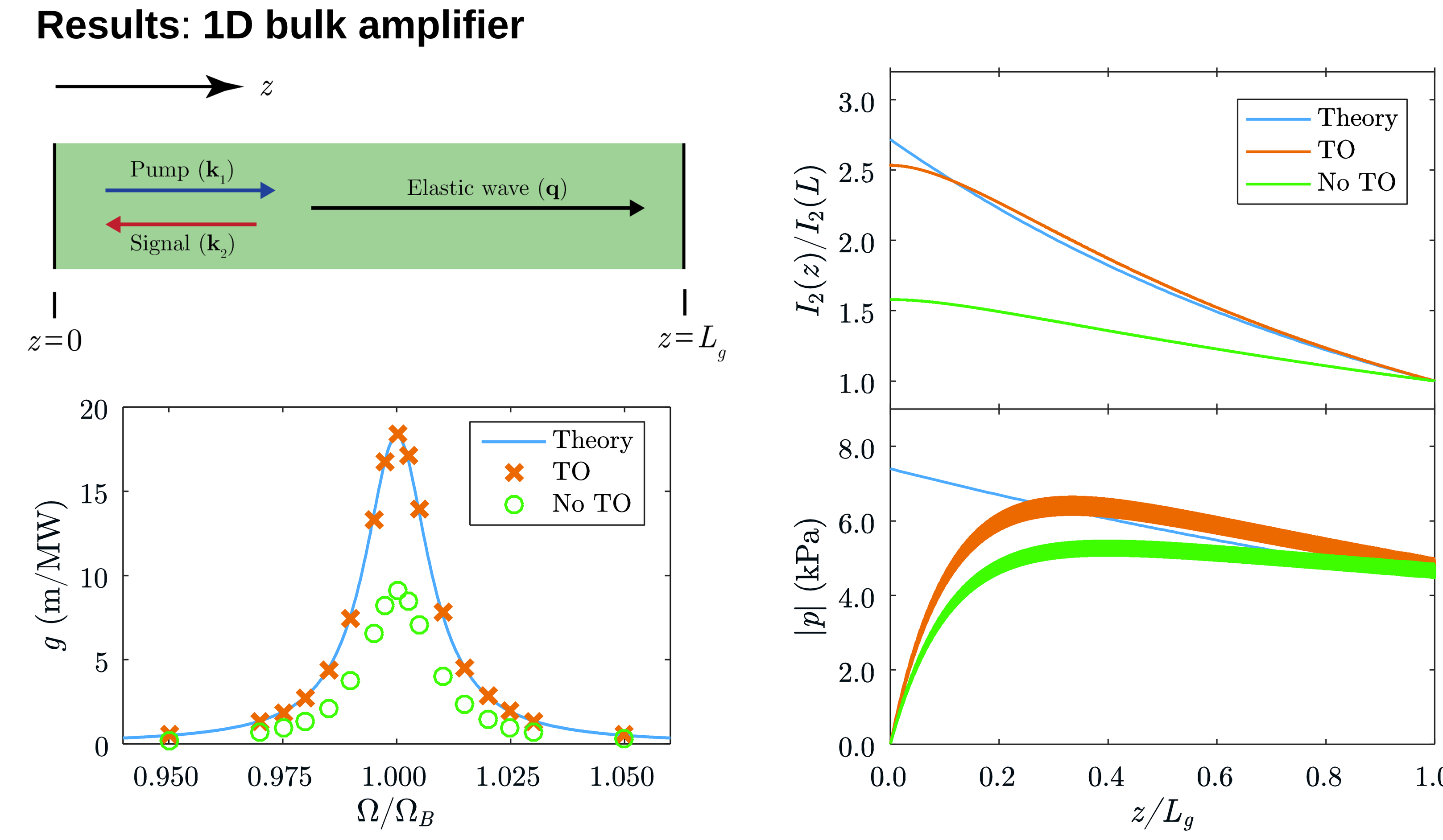

Figure 3. (top left) diagram of 1D bulk amplifier; comparison of theory [1] and simulations: (bottom left) gain spectrum, (right) relative signal intensity, elastic pressure.

Results: **slab waveguide amplifier**

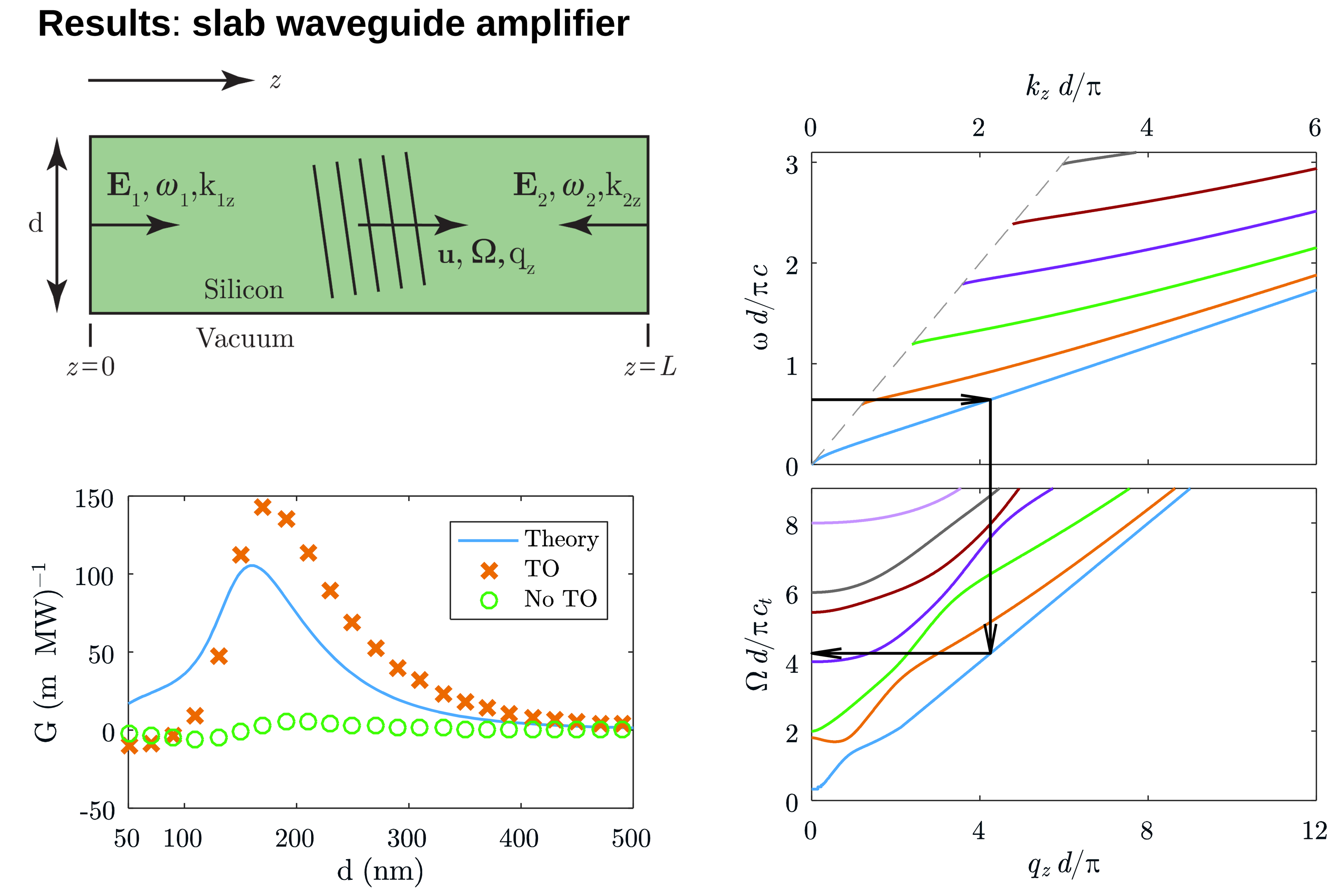

Figure 4. (top left) diagram of slab waveguide amplifier; (right) example of selection rules (momentum conservation); (bottom left) comparison of theory [4] and simulations: gain as a function of waveguide thickness.

Conclusions: The method accurately predicts SBS phenomena. It can work with any geometry and incorporate arbitrarily refined descriptions of optical forces, without requiring prior analytic knowledge of the problem. Future developments may include extending applicability to anisotropic materials and fluid domains, and designing of artificial media, in particular metamaterials and plasmonic systems.

References:

R. Zecca *et al.*, Phys. Rev. A (2016) (in review)
1. R. W. Boyd, *Nonlinear Optics*, 3rd ed., ch. 9 (Academic Press, 2008).
2. P. T. Rakich *et al.*, Phys. Rev. X **2**, p. 011008 (2012).
3. W. Qiu *et al.*, Opt. Expr. **21** p. 31402 (2013).
4. C. Wolff *et al.*, Phys. Rev. A **92,** p. 013836 (2015).
5. A. J. Ward and J. B. Pendry, J. Mod. Opt. **43**, p. 773 (1996).
6. U. Leonhardt, Science **312**, p. 1777 (2006).
7. J. B. Pendry *et al.*, Science **312**, p. 1780 (2006).