

# Photon Migration through Multiple Layers of Biological Tissue

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## INTRODUCTION

Optical tomography involves the non-invasive measurement of photon flight through soft tissue. Through the reconstruction of images made from the transmitted light & scatter, the study of photon paths through soft tissue can be used to assess anatomical structures & tissue under investigation. High scatter-based attenuation is frequently performed using intense, often pulsed or modulated light sources & at frequencies where body tissues are most transmissive. The optical properties of tissue vary considerably over a range of wavelengths, soft tissues are highly scattering but inadequately absorb in the near-infrared range, thus this wavelength is typically used. The separation of absorption from scatter is done with either time-resolved or frequency domain data which is then fitted with a diffusion theory of how light propagates through the tissue. The measurement of time of flight or frequency domain phase shift is essential to allow separation of absorption from scatter with sufficient accuracy.

## AIM

To produce a multilayer photon migration model to predict the MFP on which photons will be found can be obtained from the path of the net flux propagation using the diffusion equation. The diffusion equation is valid when studying diffuse light propagation. The modelling of light propagation through multiple layers of biological tissue are assessed & compared to the theoretical predictions by Perelman *et al.* [94 & 95] for the most-favourable-path (MFP) as described by Equation (1) & analytical solutions for light propagation phase ( $\phi$ ), lag ( $\phi_{lag}$ ), amplitude of attenuation ( $A_{att}$ ) as described by Equations (2)-(4), & illustrated in Figure 1 below.

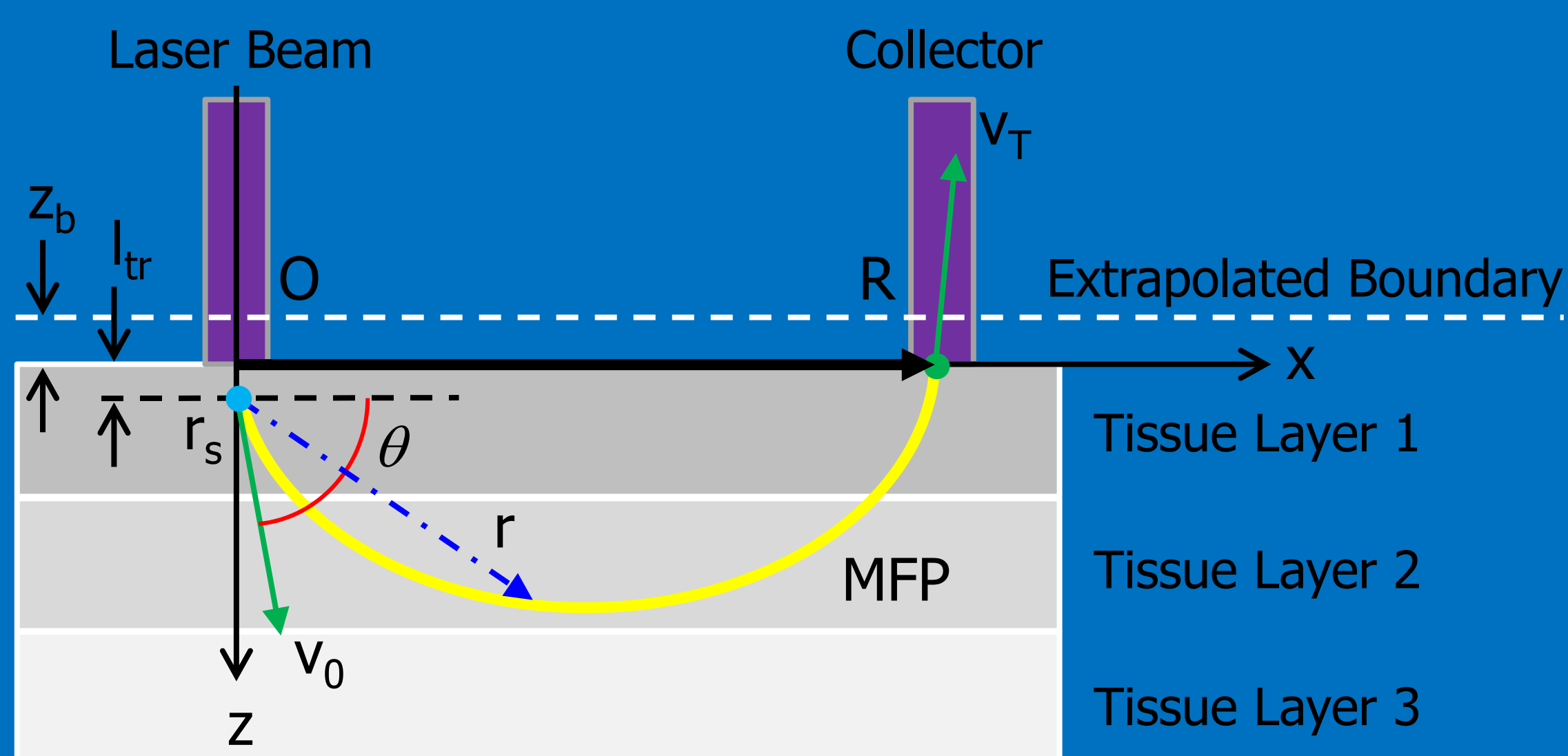


Figure 1. Schematic diagram of experimental geometry & coordinates

**Equation 1.** General form of MFP obtained from Perelman *et al* [95]

$$r_{cl}(t) = \frac{(v_T + v_0)T - 2R}{T^3} t^3 + \frac{3R - (v_T + 2v_0)T}{T^2} t^2 + v_0 t \quad (1)$$

**Equations 2.** Analytical solutions obtained from Pham *et al* [2000]

$$\phi(r, t) = \frac{A_{dc}}{4\pi D} \times \frac{\exp(-r/\delta)T}{r} + \frac{A_{ac}}{4\pi D} \times \frac{\exp(-k_{real} \times r)}{r} \quad (2)$$

$$\Theta_{lag}(\rho, \omega) = \frac{(v_T + v_0)T - 2R}{T^3} k_{imag}(\omega) r_0 - \arctan\left(\frac{IMAG}{REAL}\right) \quad (3)$$

$$A_{att}(\rho, \omega) = \frac{A_{tr}}{4\pi D} (REAL^2 + IMAG^2)^{\frac{1}{2}} \quad (4)$$

where:

$$k_{real} = \sqrt{\frac{3}{2} \mu_a \mu_s' \left\{ 1 + \left( \frac{\omega}{c \mu_a} \right)^{2/3} + 1 \right\}^{1/2}}$$

$$k_{imag} = \sqrt{\frac{3}{2} \mu_a \mu_s' \left\{ 1 + \left( \frac{\omega}{c \mu_a} \right)^{2/3} - 1 \right\}^{1/2}}$$

$$r_0 = [(\mu_s')^{-2} + \rho^2]^{1/2}$$

$$r_{ob} = \left[ \left( \frac{4}{3} \frac{1 + R_{eff}}{\mu_s'} + \frac{1}{\mu_s'} \right)^2 + \rho^2 \right]^{1/2}$$

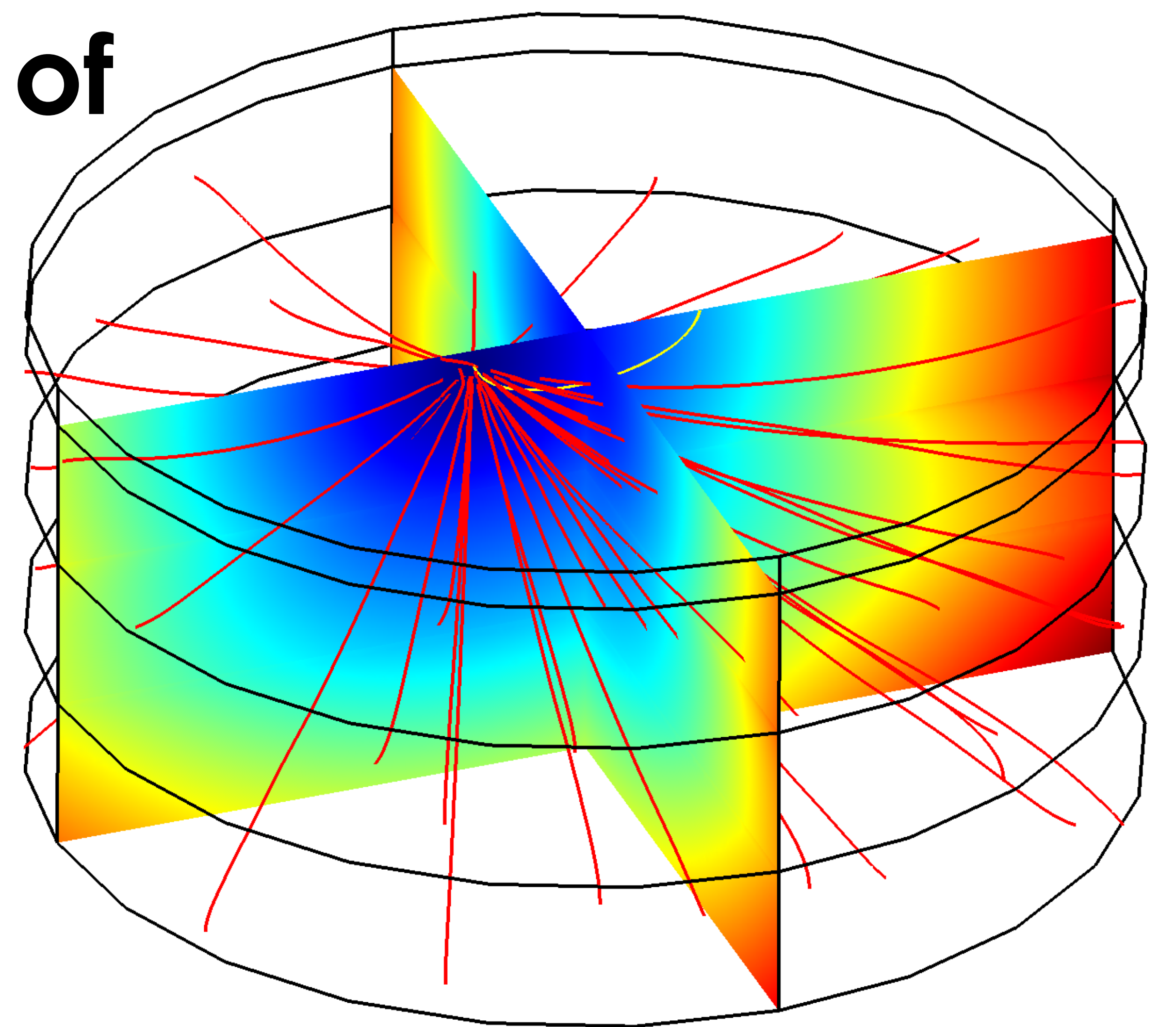
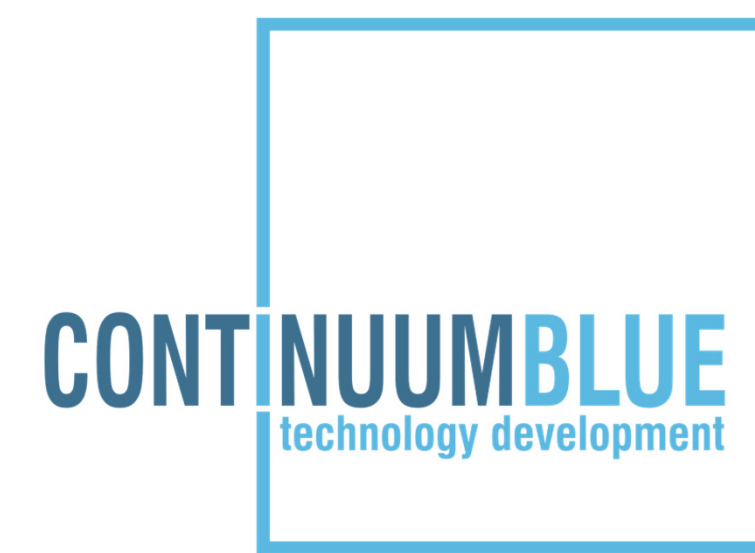
$$REAL = \frac{\exp[-k_{real}(\omega)r_0]}{r_0} - \cos[k_{imag}(\omega)(r_{ob} - r_0)] \frac{\exp[-k_{real}(\omega)r_{ob}]}{r_{ob}}$$

$$IMAG = \sin[k_{imag}(\omega)(r_{ob} - r_0)] \frac{\exp[-k_{real}(\omega)r_{ob}]}{r_{ob}}$$

## References:

- Zhou & Bai, Photon migration in turbid media: a finite-element solution for the most favorable path, *Opt. Eng.* 41(10) 2577–2581 (2002)
- Perelman *et al.*, Photon migration in turbid media using path integrals, *Phys. Rev. Lett.* 72(9), 1341–1344 (1994)
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- Pham *et al.*, Broad Bandwidth Frequency Domain Instrument for Quantitative Tissue Optical Spectroscopy, *Rev. Sci. Instrum.* 71 (6), (2000)
- Fishkin *et al.*, Frequency-domain photon migration measurements of normal and malignant tissue optical properties in a human subject, *Applied Optics* 36(1), (1997)

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## METHOD

By using the diffusion equation as described by Equation (5), making  $\phi(r, \omega) = u$ , & implementing this in to the Helmholtz Equation to describe diffusion as described by Equation (6) & multiplying all terms by  $-D$ , one obtains Equation (7). For tissue without a fluorescent source,  $f = 0$  in Equation (6).

$$(\nabla^2 + k^2) \phi(r, \omega) = -\frac{1}{D} S(r, \omega) \quad (5)$$

$$-\nabla \cdot (c \nabla u) + au = f \quad (6)$$

$$-\nabla(D \nabla u) - Dk^2 u = S(r, \omega) \quad (7)$$

Where for a layer of tissue:  $c = D$

$$a = -Dk^2$$

$$f = S(r, \omega)$$

## OBSERVED COEFFICIENTS

Fishkin *et al* [97] obtained relationships for absorption coefficient ( $\mu_a$ ) & scattering coefficient ( $\mu_s$ ) with respect to wavelength ( $\lambda$ ) for both tumor & non-tumor human tissue. Additionally, Pham *et al.* [2000] used reduced scattering values calculated from previously reported intralipid optical properties, where Mie theory was used to relate  $\mu_s$  & the anisotropy factor ( $g$ ) to the optical wavelength for 10% intralipid, as given by Equations (8) & (9):

$$\mu_s(\lambda) = 16\lambda^{-2.4} \quad (8)$$

$$g(\lambda) = 1.1 - 0.58\lambda \quad (9)$$

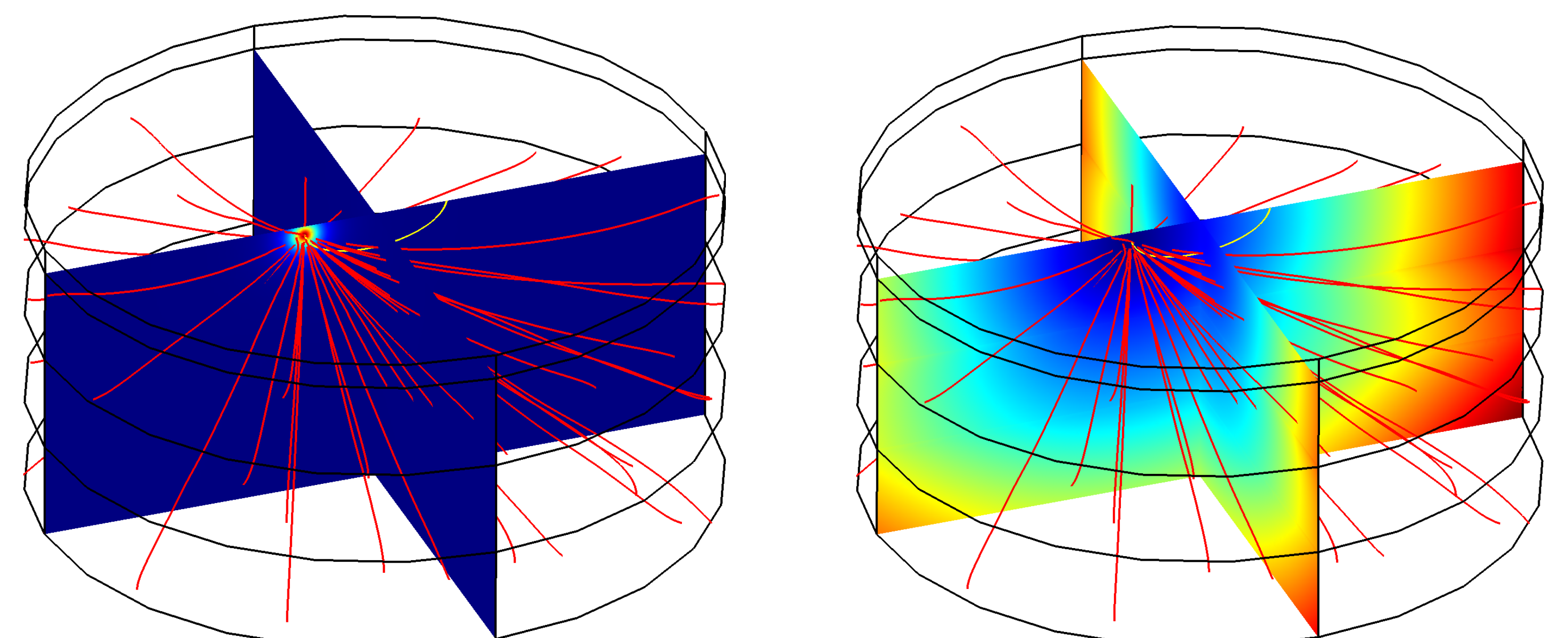


Figure 2. Phase lag & relative amplitude plots through normal & tumour scattering & absorption coefficients obtained from Fishkin *et al.* [97]

## RESULTS & CONCLUSION

The MFP on which photons will be found can be obtained from the path of the net flux using the diffusion equation; Figure 2 illustrates example phase lag & relative amplitude through layers of normal & tumour tissue. The diffusion equation is valid when studying diffuse light propagation, where photon scattering is much greater than the absorption, or at a sufficient distance from the light sources. The diffusion intensity and net flux through multiple layers of biological tissue were calculated using COMSOL. Variations in model parameters such as source-detector separation, absorption coefficient, scattering coefficient & anisotropic properties were assessed & compared with the theoretical predictions of MFP.