Load Cell Design Using COMSOL Multiphysics

Andrei Marchidan, Tarah N. Sullivan and Joseph L. Palladino
Department of Engineering, Trinity College, Hartford, CT 06106, USA
joseph.palladino@trincoll.edu

***Abstract*— COMSOL Multiphysics was used to design a binocular load cell. A three-dimensional linear solid model of the load cell spring element was studied to quantify the high-strain regions under loading conditions. The load cell was fabricated from 6061 aluminum, and general purpose Constantin alloy strain gages were installed at the four high-strain regions of the spring element. The four gages were wired as a full Wheatstone bridge configuration and total strain was measured for applied loads ranging from 0-2.5 kg in 100 g increments. Model total strain was measured using point probes at each of the four strain locations, and with a load parametric analysis. Absolute mean model-predicted strain was 1.41% of measured strain. The load cell was highly linear, with correlation coefficient r^2=0.9999.**

Keywords: load cell, COMSOL Multiphysics, solid mechanics, strain gage, force transducer.

I. Introduction

LOAD cells are commonly used force transducers that convert an applied mechanical load into a voltage. Load cells typically comprise spring elements that are designed to deform with load, strain gages that vary their resistance with deformation (strain) of the spring element, and a Wheatstone bridge circuit that produces voltage proportional to strain. One popular spring element design is the binocular configuration, which is a beam with two holes and a web of beam material removed, as shown in Figure 1. The complexity of the binocular section of this beam prevents prediction of strain via simple hand calculation; hence, a COMSOL model was used to guide load cell design.

II. Methods

1) Equations: Modeling the load cell requires three equations: an equilibrium balance, a constitutive relation relating stress and strain, and a kinematic relation relating displacement to strain. Newton's second law serves as the equilibrium equation, which in tensor form is:

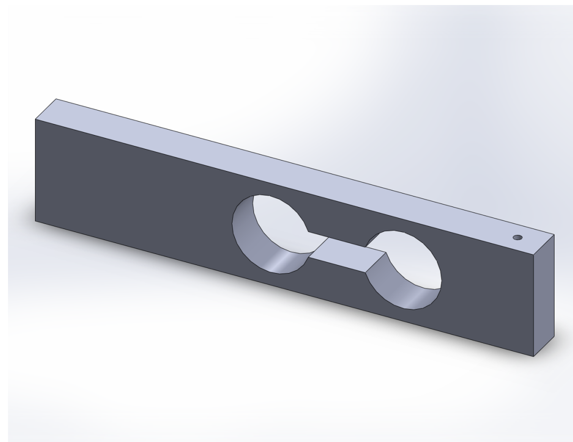

Fig. 1. Load cell spring element with "binocular" design.

$$\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{F_v} = \boldsymbol{\rho}\, \ddot{\boldsymbol{u}} \tag{1}$$

where $\boldsymbol{\sigma}$ is stress, $\boldsymbol{F_v}$ is body force per volume, $\boldsymbol{\rho}$ is density, and $\ddot{\boldsymbol{u}}$ is acceleration. For static analysis, the right-hand side of this equation goes to zero.

The constitutive equation relating the stress tensor $\boldsymbol{\sigma}$ to strain $\boldsymbol{\epsilon}$ is the generalized Hooke's law

$$\boldsymbol{\sigma} = \boldsymbol{C} : \boldsymbol{\epsilon} \tag{2}$$

where $\boldsymbol{C}$ is the fourth-order elasticity tensor and : denotes the double dot tensor product. In COMSOL, this relation is expanded to

$$\boldsymbol{\sigma} - \boldsymbol{\sigma_0} = \boldsymbol{C} : (\boldsymbol{\epsilon} - \boldsymbol{\epsilon_0} - \boldsymbol{\epsilon_{inel}}) \tag{3}$$

For this application, initial stress $\boldsymbol{\sigma_0}$, initial strain $\boldsymbol{\epsilon_0}$, and inelastic strain $\boldsymbol{\epsilon_{inel}}$ are all zero. For isotropic material, the elasticity tensor reduces to the 6X6 elasticity matrix:

$$\begin{bmatrix} 2\mu+\lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & 2\mu+\lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & 2\mu+\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \quad (4)$$

where λ and μ are the Lamé constants

$$\lambda = \frac{E\,\nu}{(1+\nu)(1-2\nu)} \quad \text{and} \quad \mu = \frac{E}{2(1+\nu)}.$$

E is elastic modulus and ν is Poisson's ratio, with material properties listed in Table I.

The final required equation is the kinematic relation between displacements $\boldsymbol{u}$ and strains $\boldsymbol{\epsilon}$. In tensor form

$$\boldsymbol{\epsilon} = \frac{1}{2}\left[\boldsymbol{\nabla u} + (\boldsymbol{\nabla u})^T\right] \quad (5)$$

where T denotes the tensor transpose. For rectangular Cartesian coordinates the strain tensor may be written in indicial notation [3]

$$\epsilon_{ij} = \frac{1}{2}\left[\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} - \frac{\partial u_\alpha}{\partial x_i}\frac{\partial u_\alpha}{\partial x_j}\right] \quad (6)$$

where α=1,2,3,.... For small deformations the higher order terms are negligible and ϵ_{ij} reduces to Cauchy's infinitesimal strain tensor:

$$\epsilon_{ij} = \frac{1}{2}\left[\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right] \quad (7)$$

TABLE I

MATERIAL PROPERTIES FOR 6061 ALUMINUM USED IN THE COMSOL MODEL [2].

Parameter	Symbol	Value
Elastic Modulus	E	69 GPa
Poisson's Ratio	ν	0.33
Density	ρ	2700 kg/m^3

2) COMSOL Multiphysics Model: A 3D solid mechanics model was built, with geometry drawn in SolidWorks (Figure 6) and imported using the COMSOL CAD Import Module. The spring element was modeled as homogeneous, linearly elastic 6061 aluminum. The left-hand end of this beam was defined as a fixed constraint boundary. Loads were applied to the right-hand end of the beam as a point load in the y direction.

A fine physics-controlled mesh was generated (Figure 2) and a stationary analysis was performed, using default solver settings. Model geometry parameters were varied until the sum of strains in all four high strain regions (defined below) provided several hundred micro-strain (1e-6) over the desired load range of 0-2.5 kg. To ensure repeatability, the four strain regions corresponding to the strain gage positions were defined as point probes, and the load was defined as a parameter that was swept over the full load range.

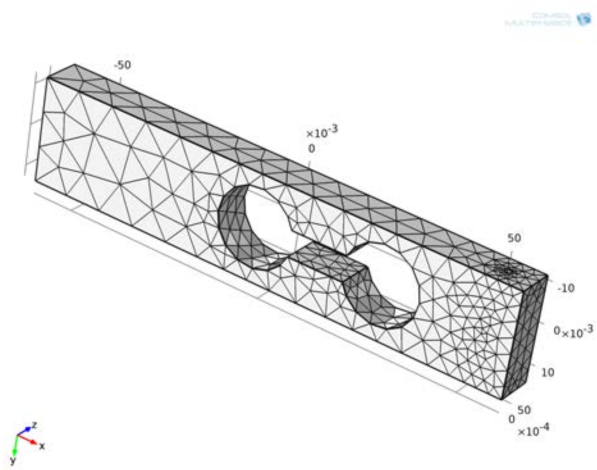

Fig. 2. COMSOL fine mesh of load cell spring element, yielding 4,885 tetrahedral elements with 23.658 degrees of freedom.

3) Model Verification: The load cell spring element was milled from 6061 aluminum bar stock and strain gages were mounted at the four high strain regions shown in Figure 3. These regions are labeled tensile strains T1 and T2 and compressive strains C1 and C2. Strain gages were general purpose CEA series polyimide encapsulated Constantan alloy (Vishay Micro-Measurements CEA-13-240UZ-120) with 120 Ω resistance and 2.2 gage factor. The gages were installed using standard surface preparation: degreasing, abrading, layout, conditioning, and neutralizing steps following the methods in [4]. They were bonded to the spring element with M-Bond 200 methyl-2-cyanoacrylate adhesive.

The four strain gages were wired with 27 AWG polyurethane insulated solid copper wire and gage

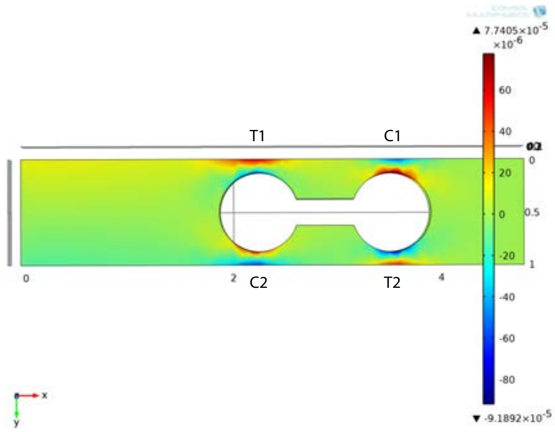

Fig. 3. Location of transducer spring element high strain regions T1, C1, T2., and C2.

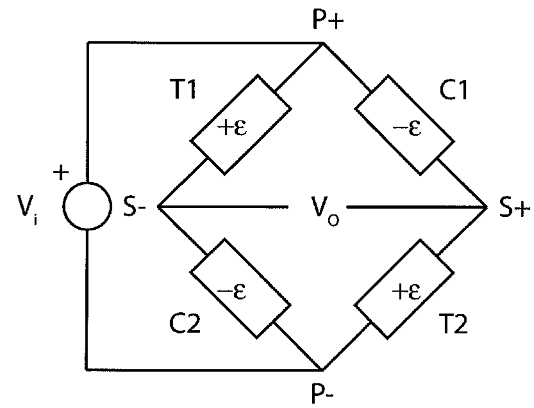

Fig. 4. Full bridge configuration of the four strain gages showing one pair of tensile and one pair of compressive strains. P+, P-, S+, and S- refer to P2 bridge amplifier connections.

lead wires were kept of uniform length to prevent unwanted lead resistance differences. The four gages were wired as a full bridge (FB 4 active) using four conductor shielded cable and connected to a bridge amplifier (Vishay P3 Strain Indicator and Recorder [5]). Figure 4 shows the full bridge layout with two opposite legs in tension and the other two in compression. Total strain is then given by

$$\epsilon_{\text{total}} = \epsilon_{\text{T1}} - \epsilon_{\text{C1}} + \epsilon_{\text{T2}} - \epsilon_{\text{C2}} \tag{8}$$

When loaded, the bridge output is linearly proportional to the load. The bridge amplifier provides the excitation voltage V_i and, after the bridge is balanced and the gage factor is input, produces an output voltage V_o directly in units of micro-strain.

III. Results

Figure 5 shows bending strain arising from an applied load of 1 kg. The highly scaled deformation image shows how the four high strain regions correspond to simultaneous tension in the T1 and T2 regions and compression in the C1 and C2 regions. Plotted is the strain tensor x-component. Table II lists strain corresponding to the locations of each of the four strain gages for each applied mass. Also shown is total model strain, computed using equation 8 in microstrain, labeled COMSOL. The next table column lists measured transducer strain, and the last column shows percent difference between model total strain and measured total strain. The absolute mean percent difference was 1.4%. Figure 7 is a plot of measured total strain as a function of applied load. The load cell is very linear, with correlation coefficient of 0.9999.

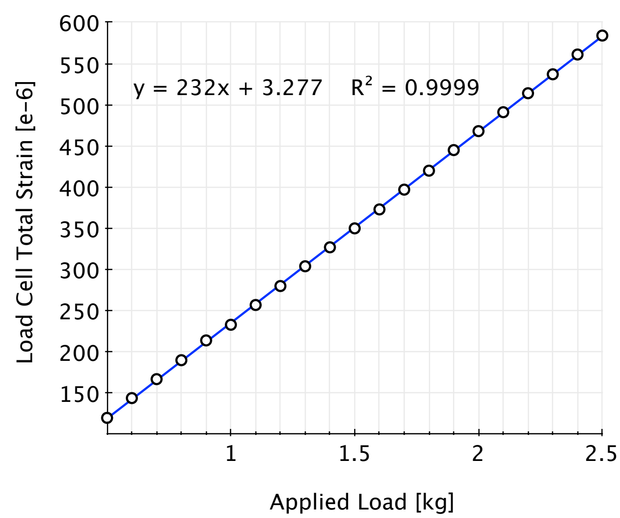

Fig. 7. Measured total strain of completed transducer for load range 0-2.5 kg, showing high degree of linearity.

IV. Discussion and Conclusions

Load cell design is challenging due to the complex geometry of spring elements. COMSOL solid models are useful for predicting strain in these designs, for locating strain gage mounting positions and especially for optimizing maximum strain for the desired load range. Model predictions were validated by measurements performed with the

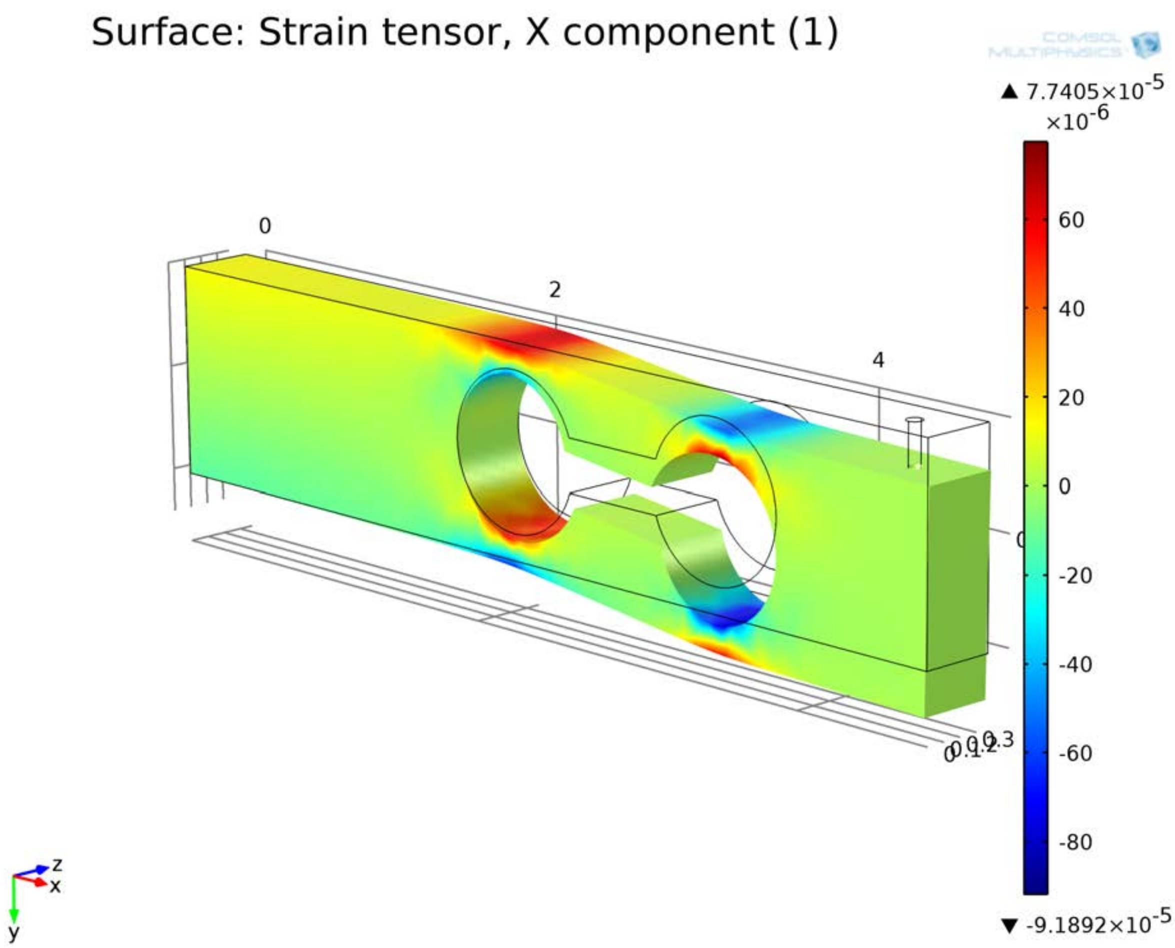

Fig. 5. Bending strain of load cell loaded with 1 kg. Exaggerated deformation shows high tensile and compressive strain regions.

completed load cell, and model and experiment agreed with absolute mean percent difference of 1.41%.

Uniformly close agreement between the COMSOL finite-element model and the measured strain demonstrates the load cell's accuracy over the design load range. Measured strains also show that this transducer is linear over the entire load range.

For maximum transducer load of 2.5 kg the maximum von Mises stress was 15 ksi, which is much less than the yield stress for 6061 aluminum (40 ksi), indicating that the transducer spring element is within the linear elastic range. Maximum displacement was 0.002 in, confirming small deformation of the spring element.

References

[1] *Strain Gage Based Transducers; Their Design and Construction*, technical report, Measurements Group Inc., Raleigh, NC, 1988.

[2] Davis, J.R. ed., *Metals Handbook Desk Edition (2ed)*, ASM International, Materials Park, OH, 1998.

[3] Fung, Y.C., *A First Course in Continuum Mechanics (2ed)*, Prentice-Hall, Englewood Cliffs, NJ, 1977.

[4] *Student Manual for Strain Gage Technology*, Bulletin 309E, Vishay Measurements Group, Raleigh, NC, 1992.

[5] *Model P3 Strain Indicator and Recorder*, instruction manual, Vishay Micro-Measurements, Raleigh, NC, 2005.

TABLE II

COMSOL MODEL STRAINS T1, C1, T2 AND C2, RESULTING MODEL TOTAL STRAIN, MEASURED TOTAL STRAIN, AND PERCENT DIFFERENCE FOR ALL APPLIED LOADS.

Mass [g]	**T1** [ϵ]	**C1** [ϵ]	**T2** [ϵ]	**C2** [ϵ]	**COMSOL** [$\mu\epsilon$]	**Measured** [$\mu\epsilon$]	**Difference** [%]
500	3.24E-05	-2.71E-05	2.69E-05	-3.26E-05	119	120	0.840
600	3.89E-05	-3.25E-05	3.23E-05	-3.92E-05	143	144	0.770
700	4.54E-05	-3.80E-05	3.76E-05	-4.57E-05	167	167	0.180
800	5.19E-05	-4.34E-05	4.30E-05	-5.22E-05	191	190	-0.262
900	5.84E-05	-4.88E-05	4.84E-05	-5.88E-05	214	214	-0.187
1,000	6.49E-05	-5.42E-05	5.38E-05	-6.53E-05	238	233	-2.183
1,100	7.14E-05	-5.96E-05	5.92E-05	-7.18E-05	262	257	-1.908
1,200	7.78E-05	-6.51E-05	6.45E-05	-7.83E-05	286	280	-1.995
1,300	8.43E-05	-7.05E-05	6.99E-05	-8.49E-05	310	304	-1.809
1,400	9.08E-05	-7.59E-05	7.53E-05	-9.14E-05	333	327	-1.920
1,500	9.73E-05	-8.13E-05	8.07E-05	-9.79E-05	357	350	-2.016
1,600	1.04E-04	-8.68E-05	8.60E-05	-1.04E-04	381	373	-2.048
1,700	1.10E-04	-9.22E-05	9.14E-05	-1.11E-04	405	397	-1.878
1,800	1.17E-04	-9.76E-05	9.68E-05	-1.18E-04	429	420	-2.189
1,900	1.23E-04	-1.03E-04	1.02E-04	-1.24E-04	452	445	-1.549
2,000	1.30E-04	-1.08E-04	1.08E-04	-1.31E-04	477	468	-1.887
2,100	1.36E-04	-1.14E-04	1.13E-04	-1.37E-04	500	491	-1.800
2,200	1.43E-04	-1.19E-04	1.18E-04	-1.44E-04	524	514	-1.908
2,300	1.49E-04	-1.25E-04	1.24E-04	-1.50E-04	548	537	-2.007
2,400	1.56E-04	-1.30E-04	1.29E-04	-1.57E-04	572	561	-1.923
2,500	1.62E-04	-1.36E-04	1.34E-04	-1.63E-04	595	584	-1.849
						Absolute Mean % Difference	1.41

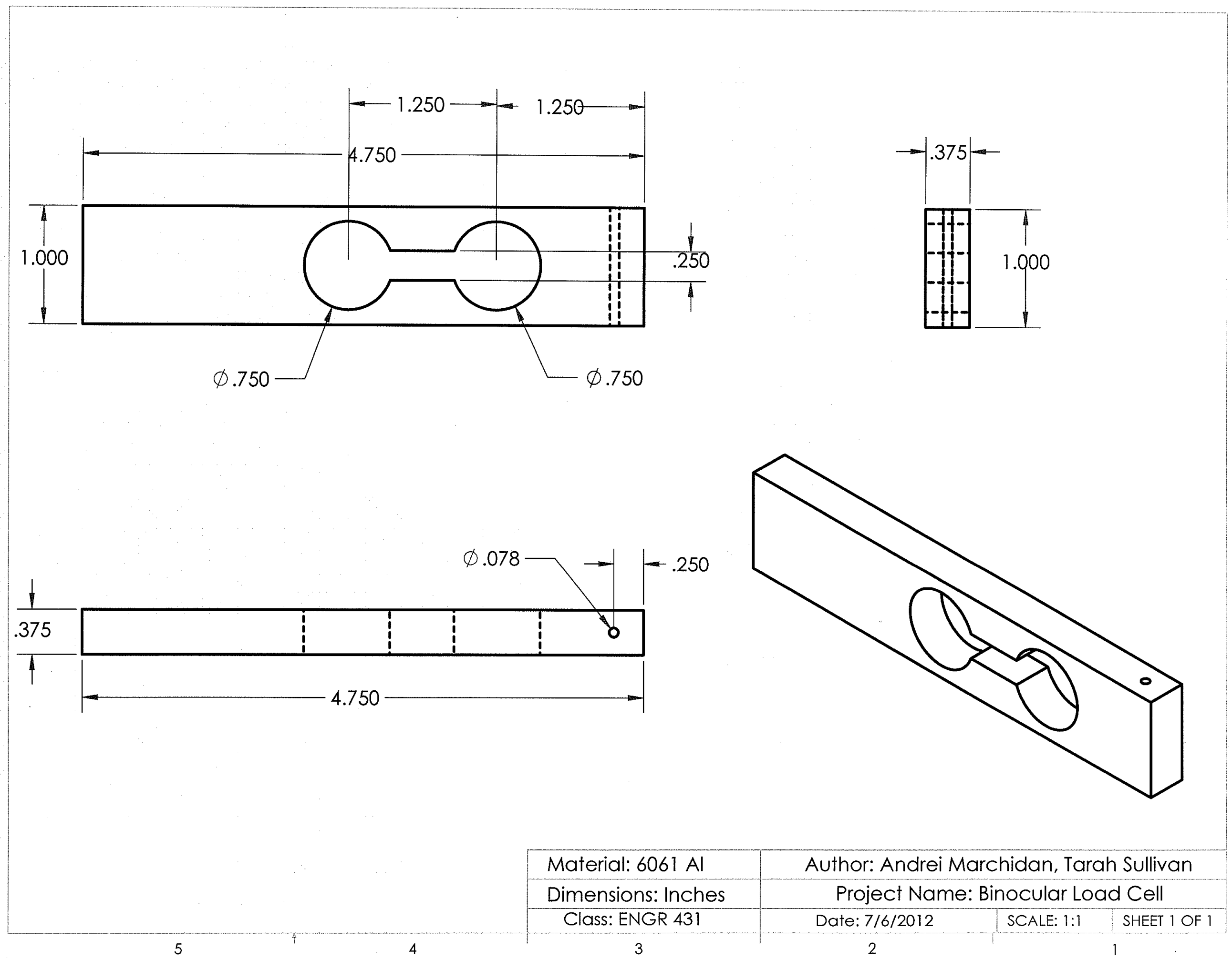

Fig. 6. Load cell geometry drawn in SolidWorks, then imported into COMSOL with CAD Import Module. The small hole at the right-hand end allows for suspending weights.